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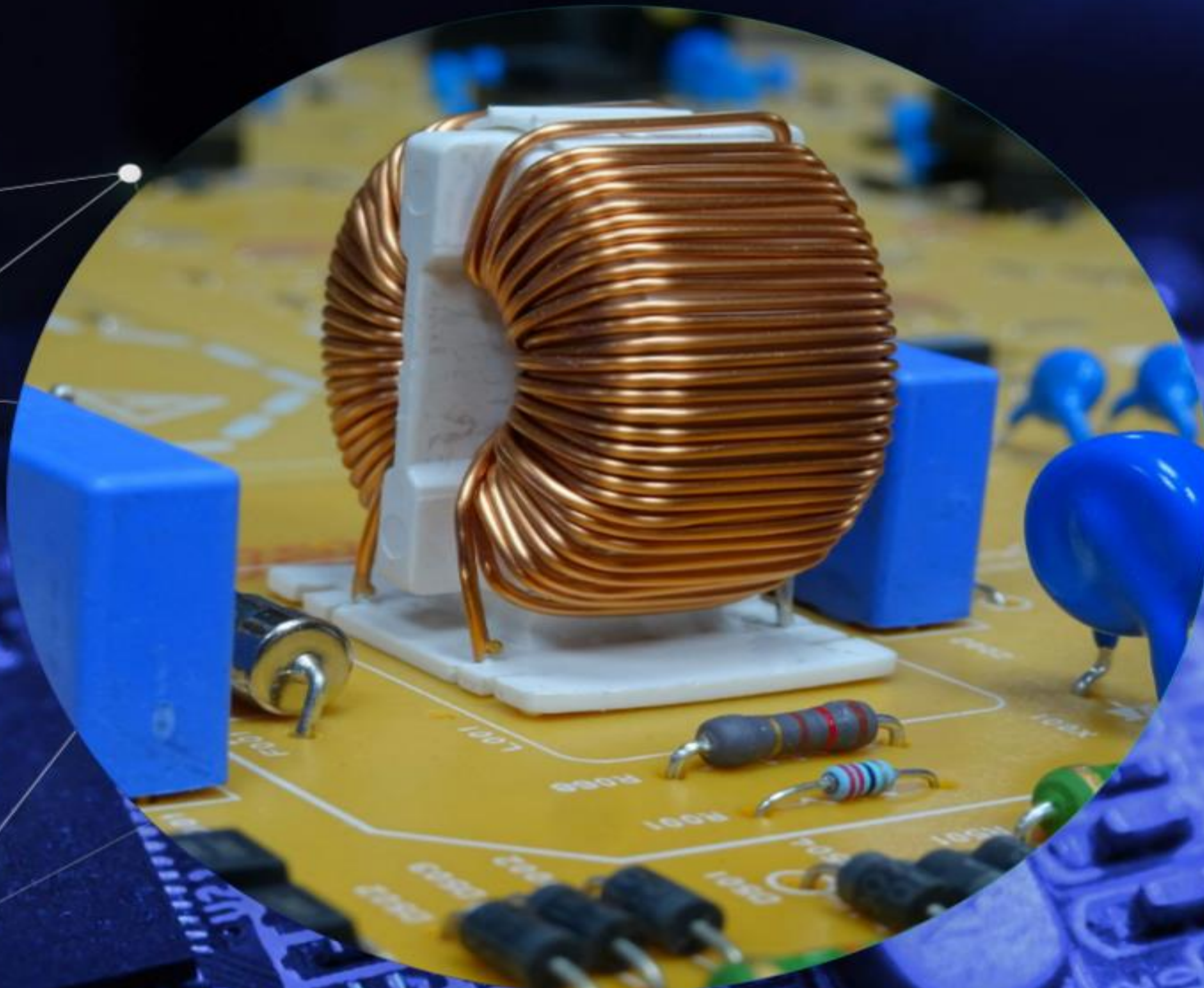

FUNDAMENTALS OF ELECTRIC CIRCUITS

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Fundamentals of Electric circuits

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1

Chapter

Basic Concepts

1.1 Introduction

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Therefore, the basic electric circuit theory course is the most important course for an electrical engineering student, and always an excellent starting point for a beginning student in electrical engineering education. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved. In electrical engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an *electric circuit*, and each component of the circuit is known as an *element.*

An electric circuit is an interconnection of electrical elements

A simple electric circuit is shown in Fig. 1.1. It consists of three basic elements: a battery, a lamp, and connecting wires. Such a simple circuit can exist by itself; it has several applications, such as a flashlight, a search light, and so forth. Electric circuits are used in numerous electrical systems to accomplish different tasks. Our objective in this book is not the study of various uses and applications of circuits. Rather our major concern is the analysis of the circuits. By the analysis of a circuit, we mean a study of the behavior of the circuit: How does it respond to a given input? How do the interconnected elements and devices in the circuit interact? We commence our study by defining some basic concepts. These concepts include charge, current, voltage, circuit elements, power, and energy. Before defining these concepts, we must first establish a system of units that we will use throughout the text.

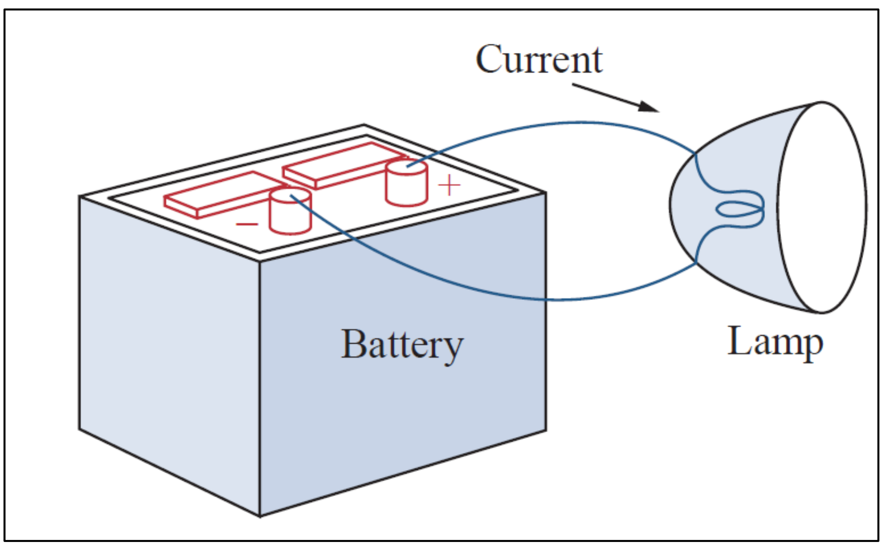


Figure 1.1 A simple Electric Circuit

1.2 Systems of Units

As electrical engineers, we deal with measurable quantities. Our measurement, however, must be communicated in a standard language that virtually all professionals can understand, irrespective of the country where the measurement is conducted. Such an international measurement language is the International System of Units (SI), adopted by the General Conference on Weights and Measures in 1960. In this system, there are six principal units from which the units of all other physical quantities can be derived. Table 1.1 shows the six units, their symbols, and the physical quantities they represent. The SI units are used throughout this text. One great advantage of the SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. Table 1.2 shows the SI prefixes and their symbols. For example, the following are expressions of the same distance in meters (m):

600,000,000 mm 600,000 m 600 km

TABLE 1.1

Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

TABLE 1.2

The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1.3 Charge and Current

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also, the most basic quantity in an electric circuit is the *electric charge*. We all experience the effect of electric charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

- Electric Charge (q):
  The electric charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (**C**). The charge e on an electron is negative and equal in magnitude to ($\mathbf{1.06 \times 10^{-19} C}$), while a proton carries a positive charge of the same magnitude as the electron.

$$\boldsymbol{q = \int_{t_1}^{t_2} i\, dt}$$

- Electric Current (i):
  The electric current is the time rate of change of charge, measured in amperes (**A**).

$$\boldsymbol{i = \frac{dq}{dt}}$$

The way we define current as *i* in Eq. (1.1) suggests that current need not be a constant-valued function. As many of the examples and problems in this chapter and subsequent chapters suggest, there can be several types of current; that is, charge can vary with time in several ways. If the current does not change with time, but remains constant, we call it a *direct current* (dc).

A direct current (dc) is a current that remains constant with time.

By convention the symbol *I* is used to represent such a constant current. A time-varying current is represented by the symbol *i*. A common form of time-varying current is the sinusoidal current or *alternating current* (ac).

An alternating current (ac) is a current that varies sinusoidally with time.

Such current is used in your household, to run the air conditioner, refrigerator, washing machine, and other electric appliances. Figure 1.2

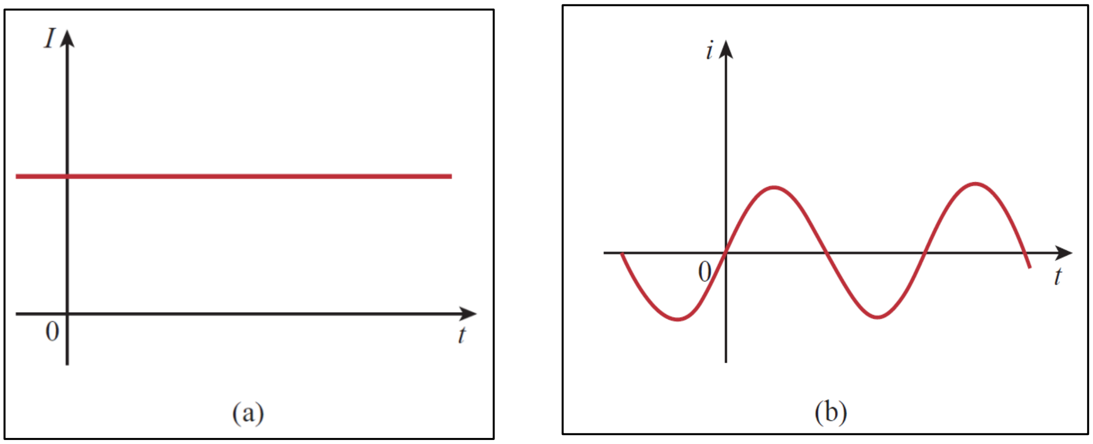


Figure 1.2 Two common types of current: (a) direct current (dc), (b) alternating current (ac).

Example 1.1

How much charge is represented by 4,600 electrons?

Solution:

Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have

$$-1.602 \times 10^{-19} \text{ C/electron} \times 4{,}600 \text{ electrons} = -7.369 \times 10^{-16} \text{ C}$$

Practice Problem 1.1

Calculate the amount of charge represented by four million protons.

Answer: $+6.408 \times 10^{-13}$ C.

Example 1.2

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

Practice Problem 1.2

If in Example 1.2, $q = (10 - 10e^{-2t})$ mC, find the current at $t = 0.5$ s.

Answer: 7.36 mA.

Example 1.3

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$Q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)\,dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

Practice Problem 1.3

The current flowing through an element is

$$i = \begin{cases} 2 \text{ A}, & 0 < t < 1 \\ 2t^2 \text{ A}, & t > 1 \end{cases}$$

Calculate the charge entering the element from $t = 0$ to $t = 2$ s.

Answer: 6.667 C.

1.4 Voltage

As explained briefly in the previous section, to move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as *voltage* or *potential difference*. The voltage V_{ab} between two points *a* and *b* in an electric circuit is the energy (or work) needed to move a unit charge from *a* to *b*; mathematically,

$$v_{ab} \triangleq \frac{dw}{dq}$$

where *w* is energy in joules (J) and *q* is charge in coulombs (C). The voltage V_{ab} or simply *v* is measured in volts (V), named in honor of the Italian physicist Alessandro Antonio Volta (1745–1827), who invented the first voltaic battery. From Eq. (1.3), it is evident that

1 volt _ 1 joule/coulomb _ 1 newton-meter/coulomb

Thus,

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

Figure 1.3 shows the voltage across an element (represented by a rectangular block) connected to points *a* and *b*. The plus (+) and minus (-) signs are used to define reference direction or voltage polarity. The *vab* can be interpreted in two ways: (1) point *a* is at a potential of *vab*

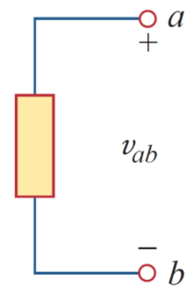


Figure 1.3 Polarity of voltage *vab*.

1.5 Power and Energy

Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves. For practical purposes, we need to know how much *power* an electric device can handle. We all know from experience that a 100-watt bulb gives more light than a 60-watt bulb. We also know that when we pay our bills to the electric utility companies, we are paying for the electric *energy* consumed over a certain period of time. Thus, power and energy calculations are important in circuit analysis.

To relate power and energy to voltage and current, we recall from physics that:

Power is the time rate of expending or absorbing energy, measured in watts (W). We write this relationship as:

$$p \triangleq \frac{dw}{dt}$$

where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s).

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$$

$$p = vi$$

Energy is the capacity to do work, measured in joules (**J**). The energy absorbed or supplied by an element from time t_0 to time **t** is:

$$w = \int_{t_0}^{t} p\, dt = \int_{t_0}^{t} vi\, dt$$

The electric power utility companies measure energy in watt-hours (Wh), where

1 Wh = 3,600 J

Example 1.4

An energy source forces a constant current of 2 A for 10 s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:
The total charge is

$$\Delta q = i\,\Delta t = 2 \times 10 = 20\text{ C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115\text{ V}$$

Practice Problem 1.4

To move charge q from point a to point b requires -30 J. Find the voltage drop v_{ab} if: (a) $q = 2$ C, (b) $q = -6$ C.

Answer: (a) -15 V, (b) 5 V.

Example 1.5

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5\cos 60\pi t\text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\,di/dt$.

Solution:
(a) The voltage is $v = 3i = 15\cos 60\pi t$; hence, the power is

$$p = vi = 75\cos^2 60\pi t\text{ W}$$

At $t = 3$ ms,

$$p = 75\cos^2(60\pi \times 3 \times 10^{-3}) = 75\cos^2 0.18\pi = 53.48\text{ W}$$

(b) We find the voltage and the power as

$$v = 3\frac{di}{dt} = 3(-60\pi)5\sin 60\pi t = -900\pi\sin 60\pi t\text{ V}$$

$$p = vi = -4500\pi\sin 60\pi t\cos 60\pi t\text{ W}$$

At $t = 3$ ms,

$$\begin{aligned} p &= -4500\pi\sin 0.18\pi\cos 0.18\pi\text{ W} \\ &= -14137.167\sin 32.4^\circ\cos 32.4^\circ = -6.396\text{ kW} \end{aligned}$$

Practice Problem 1.5

Find the power delivered to the element in Example 1.5 at $t = 5$ ms if the current remains the same but the voltage is: (a) $v = 2i$ V, (b) $v = \left(10 + 5\int_0^t i\,dt\right)$ V.

Answer: (a) 17.27 W, (b) 29.7 W.

How much energy does a 100-W electric bulb consume in two hours?

Example 1.6

Solution:

$$w = pt = 100\text{ (W)} \times 2\text{ (h)} \times 60\text{ (min/h)} \times 60\text{ (s/min)}$$
$$= 720{,}000\text{ J} = 720\text{ kJ}$$

This is the same as

$$w = pt = 100\text{ W} \times 2\text{ h} = 200\text{ Wh}$$

A stove element draws 15 A when connected to a 240-V line. How long does it take to consume 60 kJ?

Practice Problem 1.6

Answer: 16.667 s.

1.6 Circuit Elements

An electric circuit is simply an interconnection of the elements. ***Circuit analysis*** is the process of determining voltages across (or the currents through) the elements of the circuit. There are two types of elements found in electric circuits which are:

- ***Active Elements***: These elements are capable of generating energy such as generators, batteries, and operational amplifiers.
- ***Passive Elements***: These elements are incapable of generating energy such as resistors, capacitors, and inductors.

  The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources:
- ***Independent Source***: is an active element that provides a specified voltage or current that is completely independent of other circuit elements.
- ***Dependent Source or controlled source***: is an active element in which the source quantity is controlled by another voltage or current.

References:-

- Fundamentals of Electric Circuits, Charles K. Alexander, Matthew N. O. Sadiku, fourth edition.
- Electrical Circuit Theory and Technology, JOHN BIRD, Second edition.

2

Chapter

Basic Laws

2.1 Introduction

Chapter 1 introduced basic concepts such as current, voltage, and power in an electric circuit. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits. These laws, known as Ohm's law and Kirchhoff's laws, form the foundation upon which electric circuit analysis is built. In this chapter, in addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis. These techniques include combining resistors in series or parallel, voltage division, current division. The application of these laws and techniques will be restricted to resistive circuits in this chapter. We will finally apply the laws and techniques to real-life problems of electrical lighting and the design of dc meters.

2.2 Ohm's Law

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol R. The resistance of any material with a uniform cross-sectional area A depends on A and its length , as shown in Fig. 2.1(a). We can represent resistance (as measured in the laboratory), in mathematical form,

$$R = \rho \frac{\ell}{A}$$

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

That is,

$$v \propto i$$

Ohm defined the constant of proportionality for a resistor to be the resistance, R. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.)

$$v = iR$$

which is the mathematical form of Ohm's law. R is measured in the unit of ohms, designated Ω .

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

$$R = \frac{v}{i}$$

Example 2.1 An electric iron draws 2 A at 120 V. Find its resistance.

Solution:
From Ohm's law,

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

Practice Problem 2.1 The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 10 Ω at 110 V?

Answer: 11 A.

where ρ is known as the *resistivity* of the material in ohm-meters. Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities. Table 2.1 presents the values of for some common materials and shows which materials are used for conductors, insulators, and semiconductors. The circuit element used to model the current-resisting behavior of a material is the *resistor*. For the purpose of constructing circuits, resistors are usually made from metallic alloys and carbon compounds.

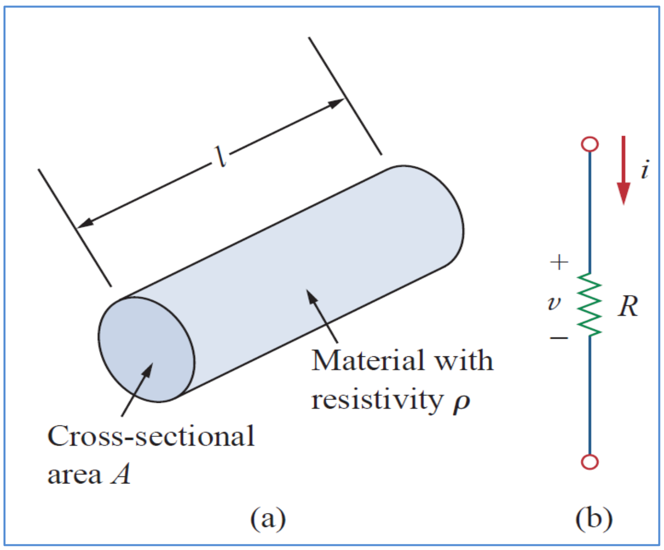


Figure 2.1 (a) Resistor, (b) Circuit symbol for

resistance.

TABLE 2.1

Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

The circuit symbol for the resistor is shown in Fig. 2.1(b), where *R* stands for the resistance of the resistor.

Conductance (G): is the ability of an element to conduct electric current; it is measured in mhos (**℧**) or Siemens (**S**).

$$G = \frac{1}{R} = \frac{i}{v}$$

The conductance is a measure of how well an element will conduct electric current.

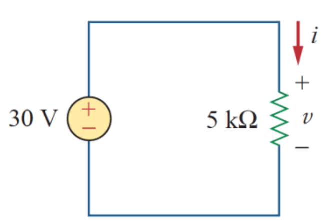
$$i = Gv$$

$$P = vi = i^2 R = \frac{v^2}{R}$$

$$P = vi = v^2 G = \frac{i^2}{G}$$

Example 2.2

In the circuit shown below, calculate the current *i*, the conductance
G, and the power *p*.



Solution:

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

or

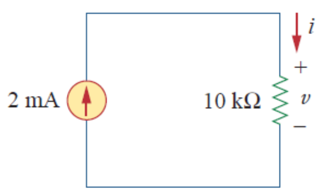
$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

Practice Problem 2.2

For the circuit shown in Fig. 2.9, calculate the voltage v, the conductance G, and the power p.



Answer: 20 V, 100 μS, 40 mW.

Example 2.3

A voltage source of $20 \sin \pi t$ V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

Solution:

$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

Hence,

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

Practice Problem 2.3

A resistor absorbs an instantaneous power of $20 \cos^2 t$ mW when connected to a voltage source $v = 10 \cos t$ V. Find i and R.

Answer: $2 \cos t$ mA, 5 kΩ.

2.3 Nodes, Branches, and Loops

Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concepts of network topology. To differentiate between a circuit and a network, we may regard a network as an interconnection of elements or devices, whereas a circuit is a network providing one or more closed paths. The convention, when addressing network topology, is to use the word network rather than circuit. We do this even though the words network and circuit mean the same thing when used in this context. In network topology, we study the properties relating to the placement of elements in the network and the geometric configuration of the network. Such elements include branches, nodes, and loops.

- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.

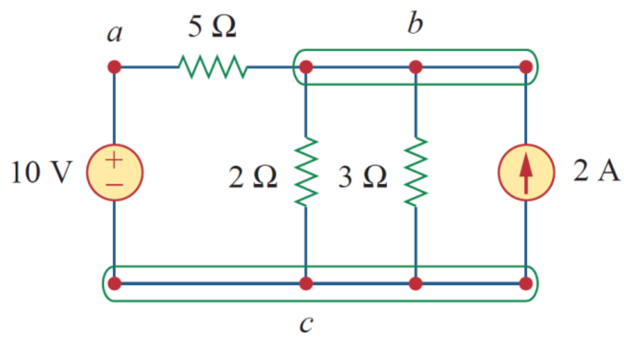


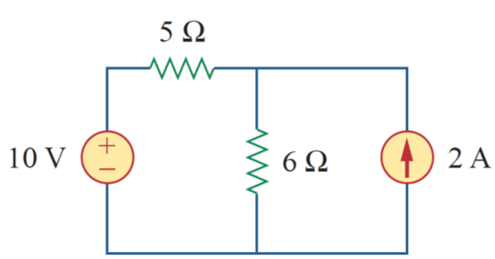
Figure 2.2 Nodes, branches, and loops.

A network with ***b*** branches, ***n*** nodes, and ***l*** independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1$$

Example 2.4

Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identify which elements are in series and which are in parallel.



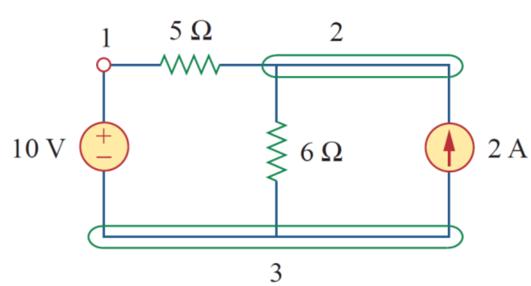


Figure 2.3 For Practice Prob. 2.4.

$l = \mathbf{2}, b = \mathbf{4}, n = \mathbf{3}$

$\boldsymbol{b = l + n - 1}$

$4 = 2 + 3 - 1 = 4 \Longrightarrow 4 = 4$

Practice Problem 2.4

How many branches and nodes does the circuit in Fig. 2.14 have? Identify the elements that are in series and in parallel.

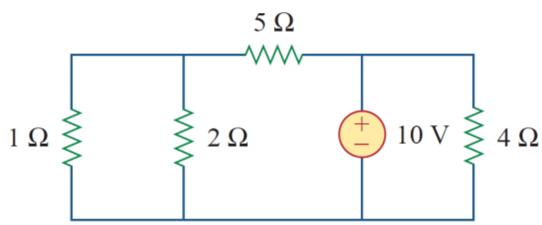


Figure 2.3 For Example 2.4.

2.4 Kirchhoff's Laws

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL). Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

- *Kirchhoff's current law (KCL)* states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that:

$$\sum_{n=1}^{N} i_n = 0$$

where $\boldsymbol{N}$ is the number of branches connected to the node and $\boldsymbol{i_n}$ is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa as shown:

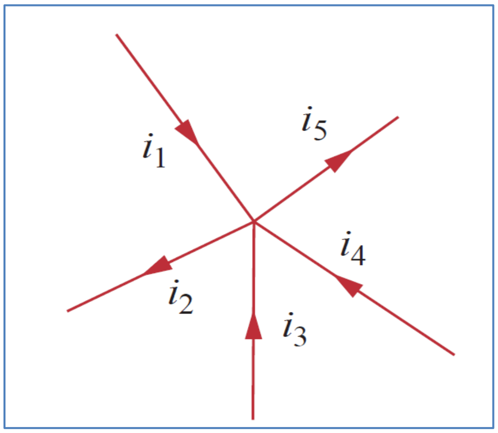


Figure 2.3 Currents at a node illustrating KCL.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

By rearranging the terms, we get:

$$i_1 + i_3 + i_4 = i_2 + i_5$$

In other word, Kirchhoff's current law states that:

(The sum of the currents entering a node is equal to the sum of the currents leaving the node).

- *Kirchhoff's voltage law (KVL)* states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that:

$$\sum_{m=1}^{M} v_m = 0$$

where ***M*** is the number of voltages in the loop (or the number of branches in the loop) and is $\boldsymbol{V_m}$ the *m*th voltage.

To illustrate KVL, consider the circuit in figure shown below.

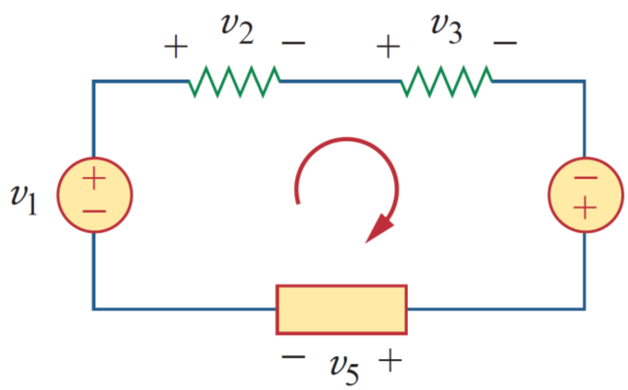


Figure 2.4 A single-loop circuit illustrating KVL.

$v_1 - v_2 - v_3 + v_4 - v_5 = 0$

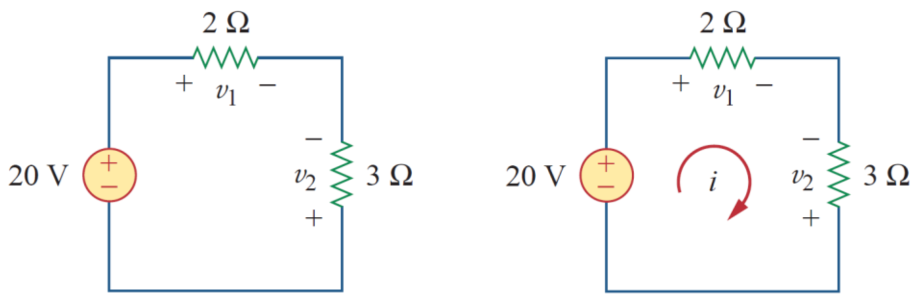
By rearranging the terms, we get:

$v_1 + v_4 = v_2 + v_3 + v_5$

This may be interpreted as:

(Sum of voltage drops = Sum of voltage rises)

Example 2.5 For the circuit in Figure find voltages v_1 and v_2.



To find $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$, we apply Ohm's law and Kirchhoff's voltage law.

$20 - v_1 - v_2 = 0 \Longrightarrow 20 - 2i - 3i = 0 \Longrightarrow 20 - 5i = 0$

$20 = 5i \Longrightarrow i = \boldsymbol{4A}$

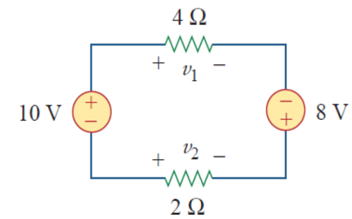
$v_1 = 2i = 2 \times 4 = \boldsymbol{8V}$ $\qquad v_2 = 3i = 3 \times 4 = \boldsymbol{12V}$

$E = v_1 + v_2 = 8 + 12 = 20V \Longrightarrow \boldsymbol{20 = 20}$

Find v_1 and v_2 in the circuit of Fig.

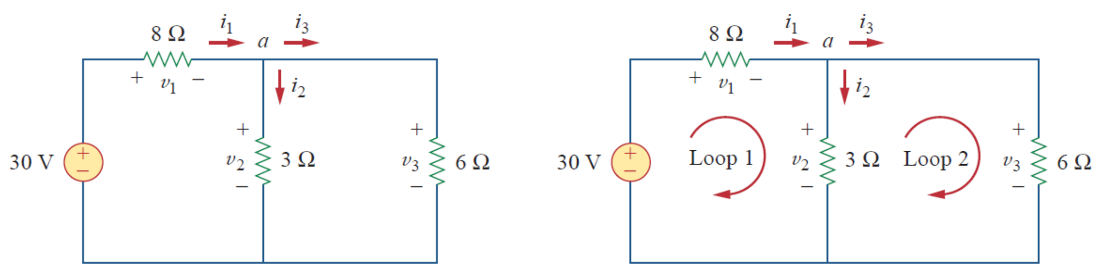
Practice Problem 2.5

Answer: 12 V, −6 V.



Example 2.8

Find currents and voltages in the circuit shown in Fig. 2.27(a).



Solution:

Using Ohm's law:

$v_1 = i_1 R_1 = 8i_1$, $v_2 = i_2 R_2 = 3i_2$, $v_3 = i_3 R_3 = 6i_3$

❖ Applying **KCL** at node a:

$i_1 = i_2 + i_3 \Longrightarrow \boxed{i_1 - i_2 - i_3 = 0}$ Eq. (1)

❖ Applying **KVL** to loop (**1**):

$E - v_1 - v_2 = 0 \Longrightarrow 30 - 8i_1 - 3i_2 = 0 \Longrightarrow \boxed{i_1 = \dfrac{30 - 3i_2}{8}}$ Eq. (2)

❖ Applying **KVL** to loop (2):

$v_2 - v_3 = 0 \Longrightarrow v_2 = v_3 \Longrightarrow 3i_2 = 6i_3 \Longrightarrow \boxed{i_3 = \dfrac{i_2}{2}}$ Eq. (3)

Substituting Eqs. (2) and (3) into (1) gives:

$\dfrac{30 - 3i_2}{8} - i_2 - \dfrac{i_2}{2} = 0$ $i_2 = \mathbf{2A}$

$i_1 = \dfrac{30 - 3(2)}{8} = \mathbf{3A},$ $i_3 = \dfrac{i_2}{2} = \dfrac{2}{2} = \mathbf{1A}$

$v_1 = 8i_1 = 8 \times 3 = \mathbf{24V}, \quad v_2 = 3i_2 = 3 \times 2 = \mathbf{6V}, \quad v_3 = 6i_3 = 6 \times 1 = \mathbf{6V}$

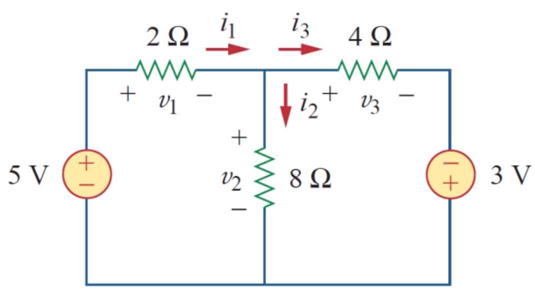
$i_1 = i_2 + i_3 = 2 + 1 = \mathbf{3A}$, **3A=3A**

$E = v_1 + v_2 = 24 + 6 = \mathbf{30V}, \quad \mathbf{30V = 30V}$

Find the currents and voltages in the circuit shown in Fig.

Practice Problem 2.6

Answer: $v_1 = 3$ V, $v_2 = 2$ V, $v_3 = 5$ V, $i_1 = 1.5$ A, $i_2 = 0.25$ A, $i_3 = 1.25$ A.



2.5 Series Resistors and Voltage Division

The need to combine resistors in series or in parallel occurs so frequently that it warrants special attention. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 2.5 The two resistors are in series, since the same current *i* flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v1 = iR1, \; v2 = iR2$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v1 + v2 = 0$$

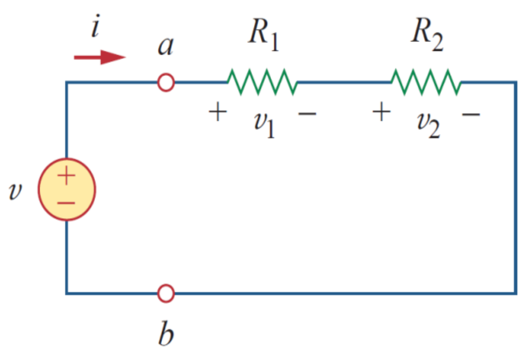


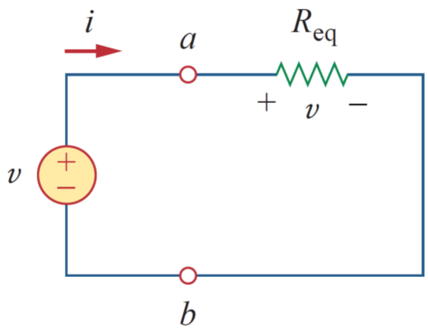
Figure 2.5
A single-loop circuit with two resistors in series.

we get $v = v1 + v2 = i\,(R_1 + R_2)$

$$i = \frac{v}{R_1 + R_2}$$ or can be $$v = iR_{eq}$$ written as

implying that the two resistors can be replaced by an equivalent resistor R_{eq}; that is,

$$R_{eq} = R_1 + R_2$$



Thus, Fig. 2.4 can be replaced by the equivalent circuit in Fig. 2.5. The two circuits in Figs. 2.4 and 2.5 are equivalent because they exhibit the same voltage-current relationships at the terminals *a-b*. An equivalent circuit such as the one in Fig. 2.5 is useful in simplifying the analysis of a circuit. In general,

Figure 2.5
Equivalent circuit of the Fig. 2.4 circuit.

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

To determine the voltage across each resistor in Fig. 2.5,

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the *principle of voltage division*, and the circuit in Fig. 2.29 is called a *voltage divider*. In general, if a voltage divider has N resistors ($R1$, $R2$, . . . , RN) in series with the source voltage v, the nth resistor (R) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

2.6 Parallel Resistors and Current Division

Consider the circuit in Fig. 2.31, where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

$$v = i_1 R_1 = i_2 R_2$$

Or

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

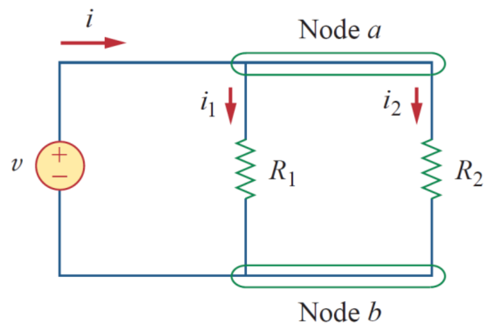


Figure 2.6
Two resistors in parallel.

Applying KCL at node *a* gives the total current *i* as

$$i = i_1 + i_2$$

Substituting Eq., we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

Where R_{eq} is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$v = iR_{eq} = \frac{iR_1 R_2}{R_1 + R_2}$$

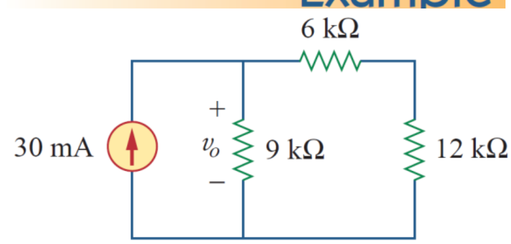
Combining Eqs. results in

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

Example

For the circuit shown, find:



a. The total resistance.
b. The total voltage.
c. The current for each resistance.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source and compare it with the total power dissipated by the resistive elements.

Solution:

a. $R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{9\times18}{9+18} = \boldsymbol{6k}\Omega$

b. $E = V = iR_T = 30 \times 10^{-3} \times 6 \times 10^{3} = \mathbf{180V} = v_1 = v_2$

c. $i_1 = \frac{v}{R_1} = \frac{180}{9\times10^3} = \boldsymbol{20mA}, \qquad i_2 = \frac{v}{R_2} = \frac{180}{18\times10^3} = \boldsymbol{10mA}$

$$i_0 = i_1 + i_2 = 20 + 10 = \boldsymbol{30A}$$

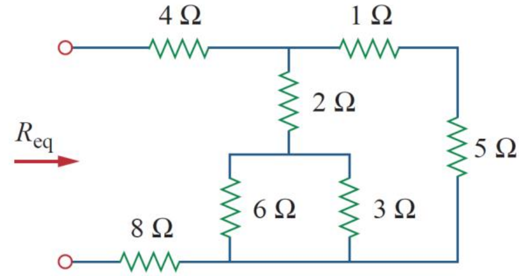
d. $P_1 = i_1 v_1 = 20 \times 10^{-3} \times 180 = 3600 \times 10^{-3} = \boldsymbol{3.6W}$

$P_2 = i_2 v_2 = 10 \times 10^{-3} \times 180 = \boldsymbol{1.8W}$

e. $P_T = i_0 \times V = 30 \times 10^{-3} \times 180 = \boldsymbol{5.4W}$

$$P_T = P_1 + P_2 = 3.6 + 1.8 = \boldsymbol{5.4W}$$

Example

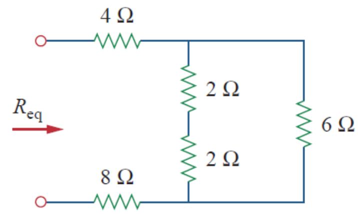


Solution

$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

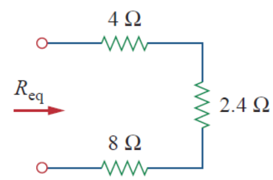
$$2\Omega + 2\Omega = 4\ \Omega$$



(a)

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

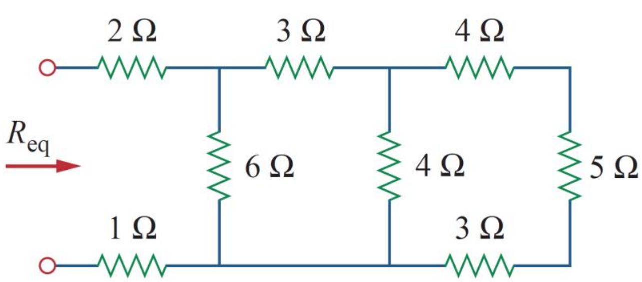


(b)

Practice Problem

By combining the resistors in Figure. find R_{eq}

Answer: 6 Ω



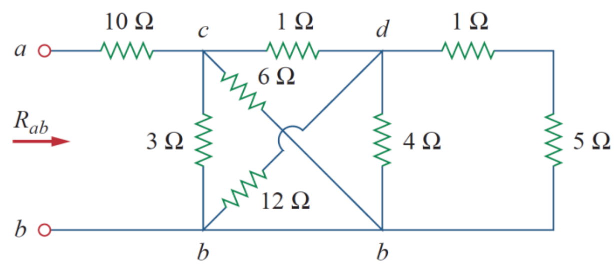
.

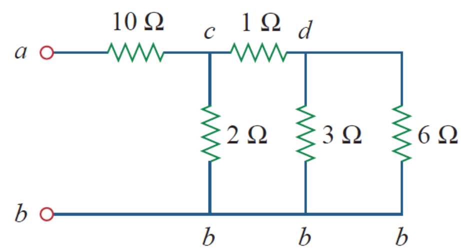
Example

Calculate the equivalent resistance R_{ab} in the circuit in Fig.

Solution

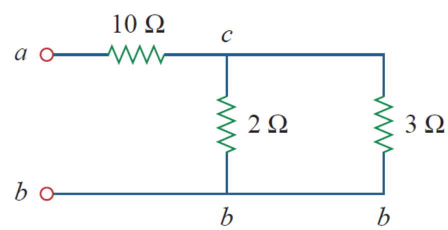
$$1+5=6\Omega,\quad \frac{4\times 12}{4+12}=3\Omega,\quad \frac{3\times 6}{3+6}=2\Omega,\quad 2+1=3\Omega$$





(a)

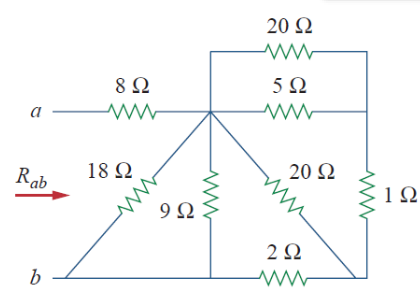
$$\frac{3\times 2}{3+2}=1.2\Omega,\qquad R_{ab}=10+1.2=\mathbf{11.2}\Omega$$



Practice Problem

Find R_{ab} for the circuit in Fig. 2.39.

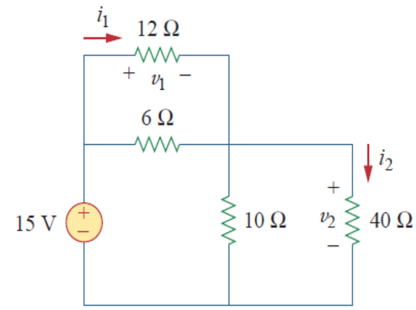
Answer: 11 Ω.



Practice Problem

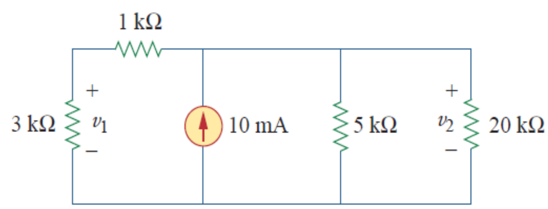
Find v_1 and v_2 in the circuit shown in Fig. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

Answer: $v_1 = 5$ V, $i_1 = 416.7$ mA, $p_1 = 2.083$ W, $v_2 = 10$ V, $i_2 = 250$ mA, $p_2 = 2.5$ W.

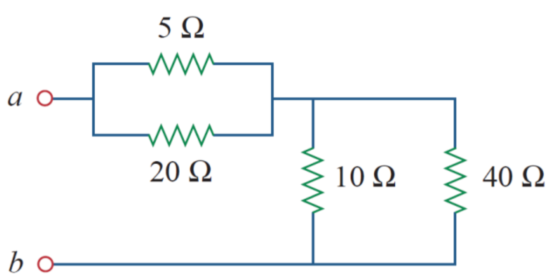


Practice Problem

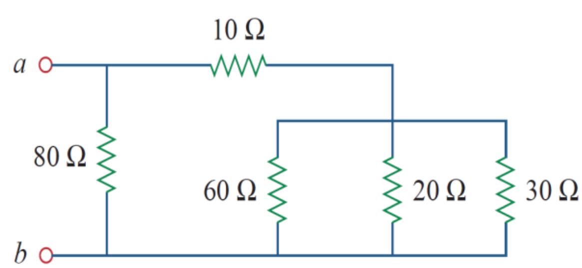
For the circuit shown in Fig. find: (a) v_1 and v_2, (b) the power dissipated in the 3-kΩ and 20-kΩ resistors, and (c) the power supplied by the current source.



Calculate the equivalent resistance *Rab* at terminals *a-b* for each of the circuits in Fig. (a,b) shown below

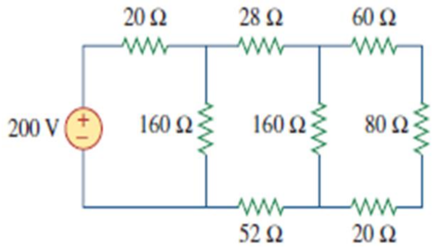


(a)



(b)

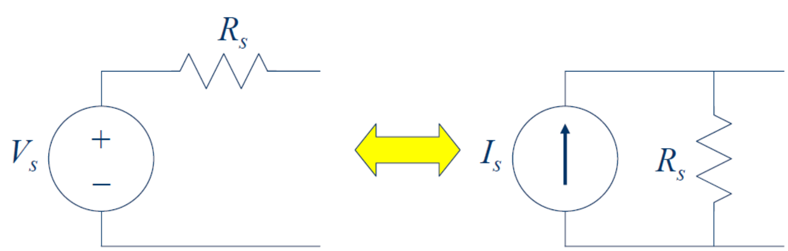
Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit of Fig. shown. Find the overall absorbed power by the resistor network.



Answer: 100Ω, P=400W.

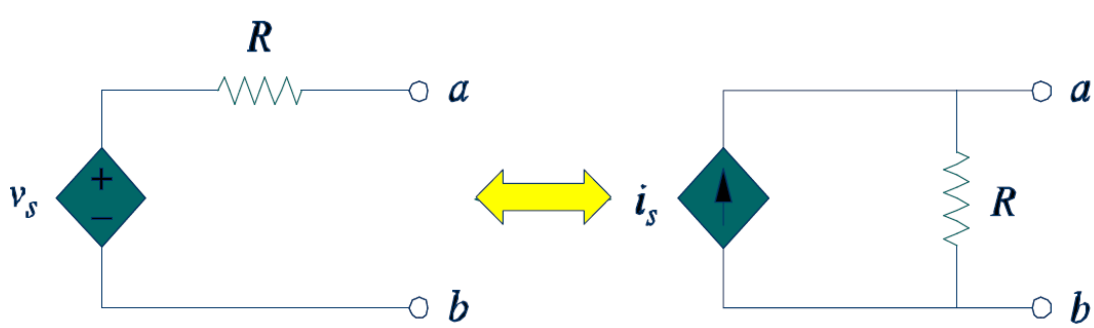
2.7 Source Transformation

- Equivalent sources can be used to simplify the analysis of some circuits.
- A voltage source in series with a resistor is transformed into a current source in parallel with a resistor.
- A current source in parallel with a resistor is transformed into a voltage source in series with a resistor.

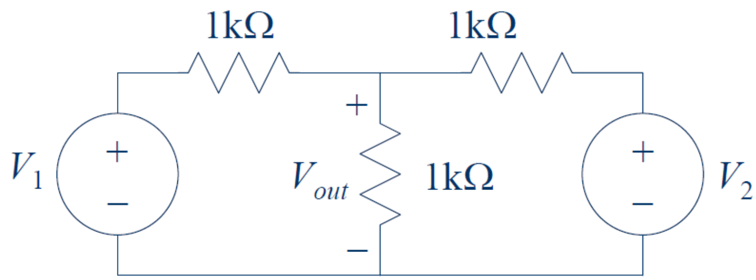


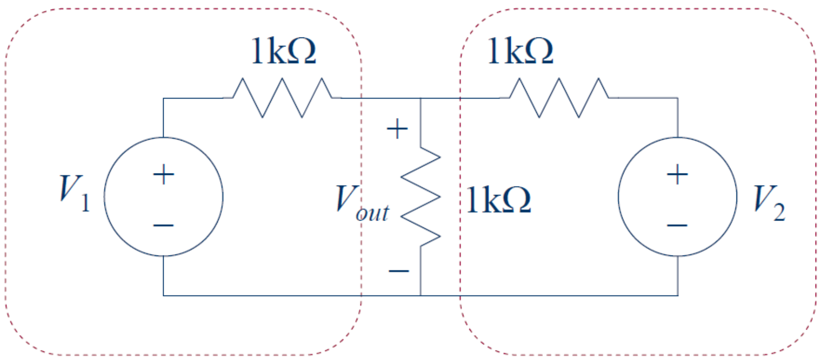
$V_s = R_s I_s$

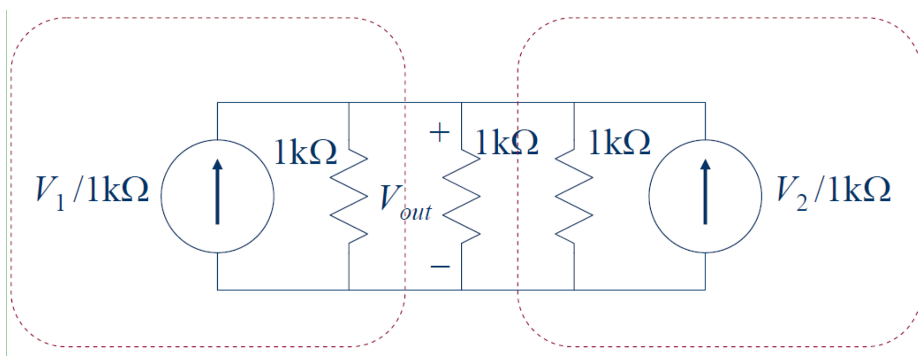
$I_s = \frac{V_s}{R_s}$



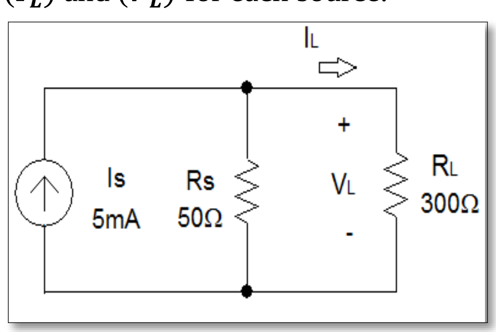
How can source transformation make analysis of this circuit easier?







Example: Convert the current source to a voltage source in the circuit shown, then calculate the current through the load ($\boldsymbol{I_L}$) and ($\boldsymbol{V_L}$) for each source.



Solution:

For the current source circuit:

$$I_L = I_s\left(\frac{R_s}{R_s + R_L}\right) = \frac{5 \times 10^{-3} \times 50}{50 + 300} = \mathbf{0.714mA}$$

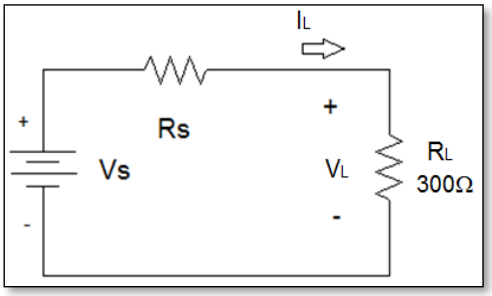
$$V_L = I_L \times R_L = 0.714 \times 10^{-3} \times 300 = \mathbf{0.214V}$$

For the voltage source circuit:

$$V_s = I_s R_s = 5 \times 10^{-3} \times 50 = \mathbf{0.25V}$$

$$I_L = \frac{0.25}{50 + 300} = \mathbf{0.714mA}$$

$$V_L = \frac{0.25 \times 300}{50 + 300} = \mathbf{0.214V}$$



References:-

- Fundamentals of Electric Circuits, Charles K. Alexander, Matthew N. O. Sadiku, fourth edition.
- Electrical Circuit Theory and Technology, JOHN BIRD, Second edition.

3

Chapter

Methods of Analysis

3.1 Introduction

Having understood the fundamental laws of circuit theory (Ohm's law and Kirchhoff's laws), we are now prepared to apply these laws to develop two powerful techniques for circuit analysis: nodal analysis, which is based on a systematic application of Kirchhoff's current law (KCL), and mesh analysis, which is based on a systematic application of Kirchhoff's voltage law (KVL). The two techniques are so important that this chapter should be regarded as the most important in the book.

3.2 Nodal Analysis

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously. To simplify matters, we shall assume in this section that circuits do not contain voltage sources. Circuits that contain voltage sources will be analyzed in the next section.

In ***nodal analysis***, we are interested in finding the node voltages. Given a circuit with *n* nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

Nodal analysis is also known as the node-voltage method.

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages v_1, $v_2, \dots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

We shall now explain and apply these three steps. The first step in nodal analysis is selecting a node as the *reference* or *datum node*. The reference node is commonly called the *ground* since it is assumed to have zero potential. A reference node is indicated by any of the three symbols in Fig.3.1.

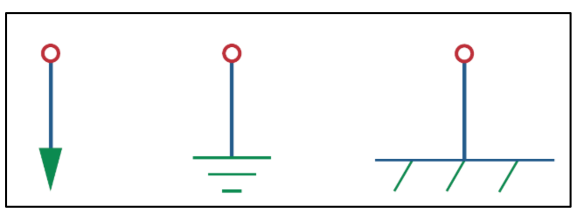
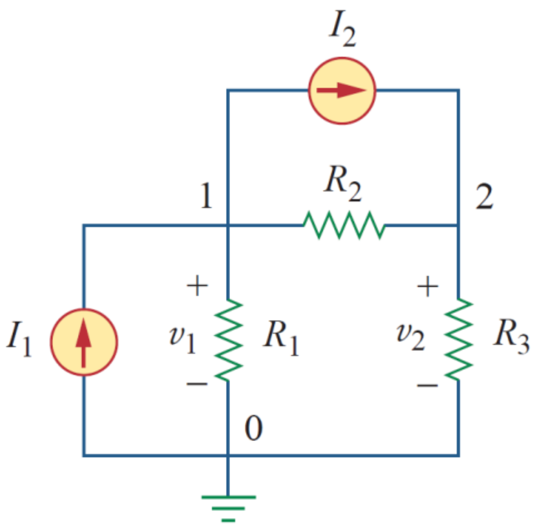
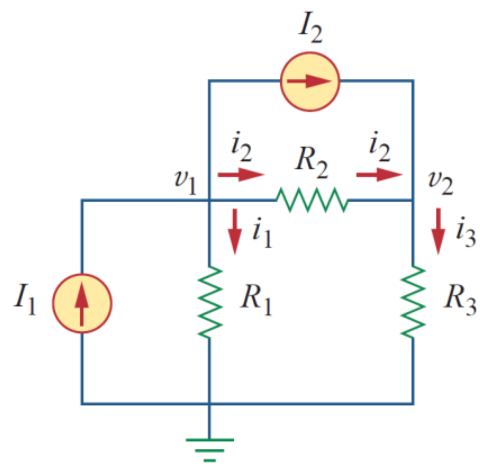


Figure 3.1
Common symbols for indicating a reference node.

Once we have selected a reference node, we assign voltage designations to nonreference nodes. Consider, for example, the circuit shown. Node **0** is the reference node while nodes 1 and 2 are assigned voltages $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ respectively.





Each node voltage is the voltage rise from the reference node to the corresponding nonreference node or simply the voltage of that node with respect to the reference node.

- The second step, we apply **KCL** to each nonreference node in the circuit. We now add $\boldsymbol{i_1}, \boldsymbol{i_2}$ and $\boldsymbol{i_3}$ as the currents through resistors $\boldsymbol{R_1}, \boldsymbol{R_2}$ and $\boldsymbol{R_3}$ respectively. At node **1**, applying **KCL** gives:

$$I_1 = I_2 + i_1 + i_2 \qquad Eq.(1)$$

At node **2:**

$$i_3 = i_2 + I_2 \qquad Eq.(2)$$

We now apply **Ohm's law** to express the unknown currents $\boldsymbol{i_1}, \boldsymbol{i_2}$ and $\boldsymbol{i_3}$ in terms of node voltages.

<u>Note: Current flows from a higher potential to a lower potential in a resistor.</u>

We can express this principle as:

$$\boldsymbol{i = \frac{v_{higher} - v_{lower}}{R}}$$

$$i_1 = \frac{v_1 - 0}{R_1}, \quad i_2 = \frac{v_1 - v_2}{R_2}, \quad i_3 = \frac{v_2 - 0}{R_3} \qquad Eq.(3)$$

Substituting Eq. (3) in Eqs. (1) and (2) results, respectively, gives:

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \qquad Eq.(4)$$

$$\frac{v_2}{R_3} = \frac{v_1 - v_2}{R_2} + I_2 \qquad Eq.(5)$$

- The third step in nodal analysis is to solve for the node voltages. If we apply **KCL** to $n-1$ nonreference nodes, we obtain $n-1$ simultaneous equations such as Eqs. (4) and (5). We solve Eqs. (4) and (5) to obtain the node voltages $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ using any standard method, such as the **substitution method**, the **elimination method**, **Cramer's rule**, or **matrix inversion**. To use either of the last two methods, one must cast the simultaneous equations in **matrix form**. For example, Eqs. (4) and (5) after we simplify them, can be cast in matrix form as:

$$v_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - v_2\left(\frac{1}{R_2}\right) = I_1 - I_2 \qquad \boldsymbol{Eq.(6)}$$

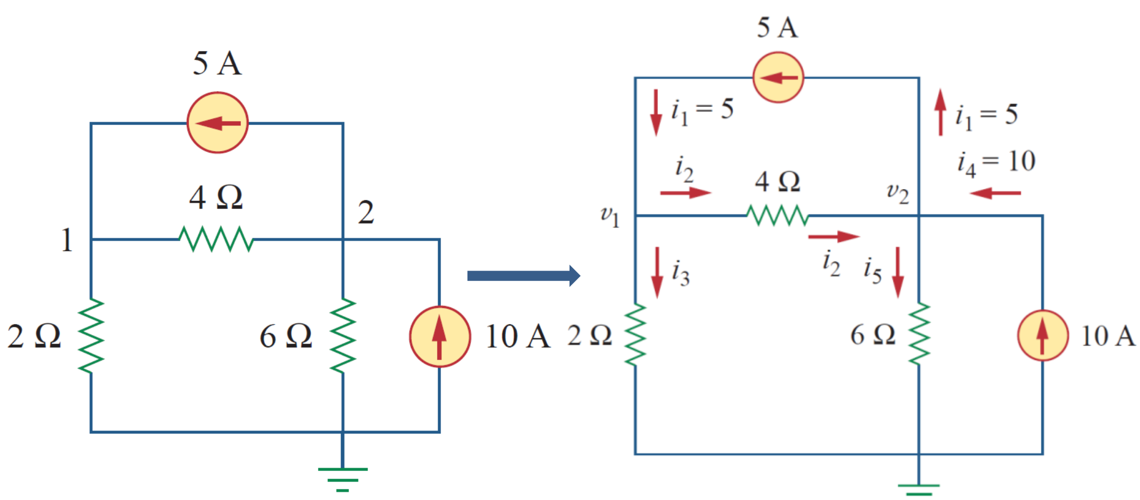
$$v_2\left(\frac{1}{R_3} + \frac{1}{R_2}\right) - v_1\left(\frac{1}{R_2}\right) = I_2 \qquad \boldsymbol{Eq.(7)}$$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2+R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Which can be solved to get $\boldsymbol{v_1}$and $\boldsymbol{v_2}$.

Example 3.1

Calculate the node voltages in the circuit shown in Fig.



For node (1):

$$\frac{v_1 - 0}{2} + \frac{v_1 - v_2}{4} = 5, \quad \frac{v_1}{2} + \frac{v_1}{4} - \frac{v_2}{4} = 5, \quad v_1\left(\frac{1}{2} + \frac{1}{4}\right) - v_2\left(\frac{1}{4}\right) = 5$$

$$v_1\left(\frac{3}{4}\right) - v_2\left(\frac{1}{4}\right) = 5, \quad \mathbf{3v_1 - v_2 = 20} \qquad \boldsymbol{Eq.(1)}$$

For node (2):

$$\frac{v_2 - v_1}{4} + \frac{v_2 - 0}{6} = 10 - 5, \quad \frac{v_2}{4} - \frac{v_1}{4} + \frac{v_2}{6} = 5, \quad v_1\left(-\frac{1}{4}\right) + v_2\left(\frac{1}{4} + \frac{1}{6}\right) = 5$$

$$v_1\left(-\frac{1}{4}\right) + v_2\left(\frac{5}{12}\right) = 5, \qquad \mathbf{-3v_1 + 5v_2 = 60} \qquad \boldsymbol{Eq.(2)}$$

We will use Cramer's rule to solve these two equations:

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} \boldsymbol{v_1} \\ \boldsymbol{v_2} \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$, We now obtain $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ as:

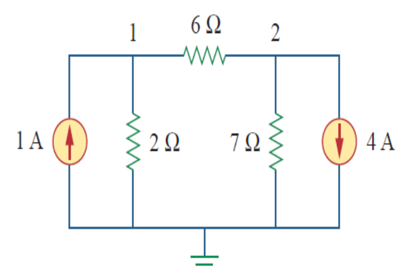
$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix}} = \frac{20 \times 5 - (-1 \times 60)}{3 \times 5 - (-1 \times -3)} = \frac{160}{12} = \mathbf{13.333V}$$

$$v_2 = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{12} = \frac{3 \times 60 - (-20 \times 3)}{12} = \frac{180 + 60}{12} = \mathbf{20V}$$

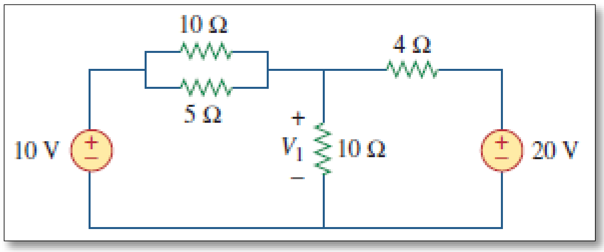
Practice Problem 3.1

Obtain the node voltages in the circuit of Fig. 3.4.

Answer: $v_1 = -2$ V, $v_2 = -14$ V.



Example 3.3 2 using nodal voltage analysis.



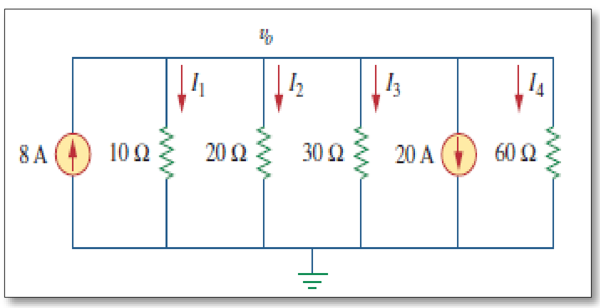
Solution:

$$\frac{v_1-10}{10}+\frac{v_1-10}{5}+\frac{v_1-20}{4}+\frac{v_1-0}{10}=0$$

$$v_1\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{4}+\frac{1}{10}\right)-1-2-5=0$$

$$v_1\left(\frac{2+4+5+2}{20}\right)=8,\quad v_1\left(\frac{13}{20}\right)=8,\quad v_1=\frac{20\times 8}{13}=\mathbf{12.307V}$$

Example 3.3 Find the currents $\boldsymbol{I_1}$ through $\boldsymbol{I_4}$ and the voltage $\boldsymbol{v_0}$ in the circuit shown using nodal voltage method.



Solution:

$$\frac{v_0-0}{10}+\frac{v_0-0}{20}+\frac{v_0-0}{30}+\frac{v_0-0}{60}=8-20$$

$$v_0\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{30}+\frac{1}{60}\right)=-12,\quad v_0\left(\frac{6+3+2+1}{60}\right)=-12$$

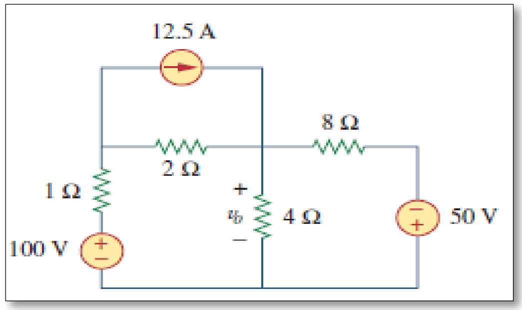
$$v_0\left(\frac{12}{60}\right)=-12,\quad v_0=\frac{-12\times 60}{12}=\mathbf{-60V}$$

$$I_1=\frac{v_0}{10}=\frac{-60}{10}=\mathbf{-6A},\qquad I_2=\frac{-60}{20}=\mathbf{-3A},\qquad I_3=\frac{-60}{30}=\mathbf{-2A},$$

$$I_4=\frac{-60}{60}=\mathbf{-1A}$$

Practice Problem 3.2

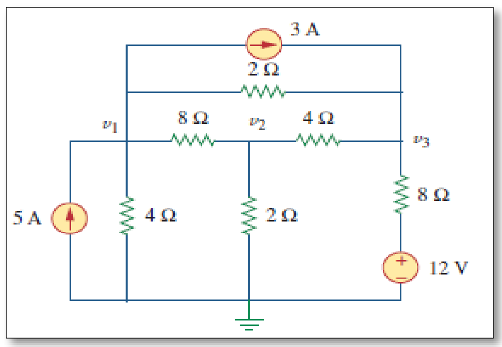
Using nodal analysis, find v_0 in the circuit shown.



Answer: $v_0 = 67.647V$

Example 3.4

Use nodal analysis to find v_1, v_2 *and* v_3 in the circuit shown.



Solution:

For node (1):

$$\frac{v_1 - 0}{4} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{2} = 5 - 3, v_1\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{2}\right) - v_2\left(\frac{1}{8}\right) - v_3\left(\frac{1}{2}\right) = 2$$

$$v_1\left(\frac{7}{8}\right) - v_2\left(\frac{1}{8}\right) - v_3\left(\frac{1}{2}\right) = 2, \qquad \mathbf{7v_1 - v_2 - 4v_3 = 16} \qquad \boldsymbol{Eq.(1)}$$

For node (2):

$$\frac{v_2 - v_1}{8} + \frac{v_2 - 0}{2} + \frac{v_2 - v_3}{4} = 0, \ \ v_2\left(\frac{1}{8} + \frac{1}{2} + \frac{1}{4}\right) - v_1\left(\frac{1}{8}\right) - v_3\left(\frac{1}{4}\right) = 0$$

$$v_1\left(\frac{-1}{8}\right) + v_2\left(\frac{7}{8}\right) - v_3\left(\frac{1}{4}\right) = 0, \quad \mathbf{-v_1 + 7v_2 - 2v_3 = 0} \qquad \boldsymbol{Eq.(2)}$$

For node (3):

$$\frac{v_3 - v_1}{2} + \frac{v_3 - v_2}{4} + \frac{v_3 - 12}{8} = 3$$

$$v_1\left(-\frac{1}{2}\right) + v_2\left(-\frac{1}{4}\right) + v_3\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) = 3 + \frac{12}{8}$$

$$v_1\left(-\frac{1}{2}\right)+v_2\left(\frac{-1}{4}\right)+v_3\left(\frac{7}{8}\right)=4.5,\qquad \mathbf{-4v_1-2v_2+7v_3=36}\qquad \boldsymbol{Eq.(3)}$$

$$\begin{bmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}=\begin{bmatrix} 16 \\ 0 \\ 36 \end{bmatrix}$$

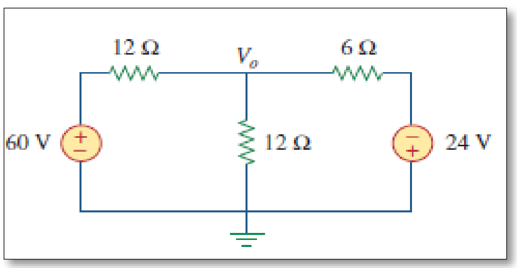
$$v_1=\frac{\Delta_1}{\Delta}=\frac{\left|\begin{array}{ccc} 16 & -1 & -4 \\ 0 & 7 & -2 \\ 36 & -2 & 7 \end{array}\right|\begin{array}{cc} 16 & -1 \\ 0 & 7 \\ 36 & -2 \end{array}}{\left|\begin{array}{ccc} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{array}\right|\begin{array}{cc} 7 & -1 \\ -1 & 7 \\ -4 & -2 \end{array}}=\frac{784+72+0+1008-64-0}{343-8-8-112-28-7}=\mathbf{10V}$$

$$v_2=\frac{\Delta_2}{\Delta}=\frac{\left|\begin{array}{ccc} 7 & 16 & -4 \\ -1 & 0 & -2 \\ -4 & 36 & 7 \end{array}\right|\begin{array}{cc} 7 & 16 \\ -1 & 0 \\ -4 & 36 \end{array}}{180}=\frac{0+128+144-0+504+112}{180}\cong\mathbf{5V}$$

$$v_3=\frac{\Delta_3}{\Delta}=\frac{\left|\begin{array}{ccc} 7 & -1 & 16 \\ -1 & 7 & 0 \\ -4 & -2 & 36 \end{array}\right|\begin{array}{cc} 7 & -1 \\ -1 & 7 \\ -4 & -2 \end{array}}{180}=\frac{1764+0+32+448-0-36}{180}$$

$$v_3=\mathbf{12.266V}$$

Example 3.5 Find the current in the **12Ω** and **6Ω** resistors using nodal voltage analysis for the circuit shown.



Solution:

There is one independent node and a reference node.

$$\frac{v_0-60}{12}+\frac{v_0-0}{12}+\frac{v_0-24}{6}=0,\quad v_0\left(\frac{1}{12}+\frac{1}{12}+\frac{1}{6}\right)-5-4=0,\quad v_0\left(\frac{1+1+2}{12}\right)=9$$

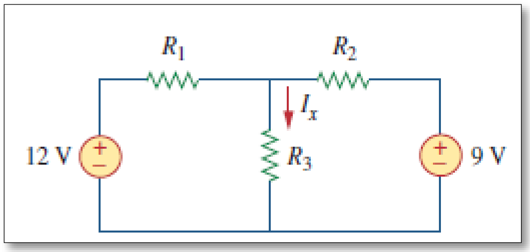
$$v_0\left(\frac{4}{12}\right)=9,\qquad v_0=\frac{12\times 9}{4}=\mathbf{27V}$$

$$I_{12\Omega}=\frac{60-v_0}{12}=\frac{60-27}{12}=\mathbf{2.75A},\quad I_{12\Omega}=\frac{v_0}{12}=\frac{27}{12}=\mathbf{2.25A}$$

$$I_{6\Omega} = \frac{24 - v_0}{6} = \frac{24 - 27}{6} = -\mathbf{0.5A}$$

Practice Problem 3.3

For the circuit shown, if $R_1 = \mathbf{1}\Omega, R_2 = \mathbf{2}\Omega, and\ R_3 = \mathbf{3}\Omega$, find $\boldsymbol{I_x}$ using nodal voltage method.



Answer: 3A.

3.3 Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Recall that a loop is a closed path with no node passed more than once. A mesh is a loop that does not contain any other loop within it. Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents. Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is *planar*.

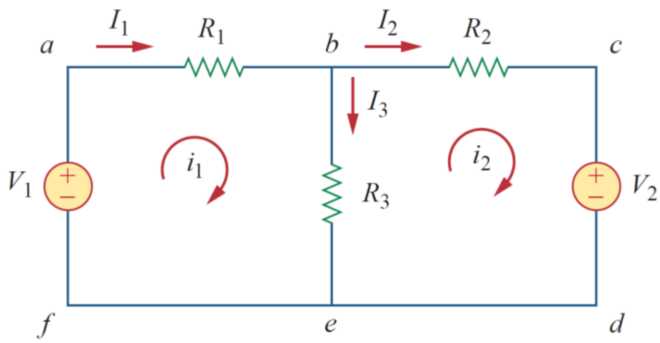
Mesh analysis is also known as loop analysis or the mesh-current method.

To understand mesh analysis, we should first explain more about what we mean by a mesh.
A mesh is a loop which does not contain any other loops within it.

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

- The first step requires that mesh currents and are assigned to meshes 1 and 2 Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.



- As the second step, we apply **KVL** to each mesh. Applying **KVL** to mesh 1, we obtain:

$$V_1 - i_1R_1 - R_3(i_1 - i_2) = 0, \;\; V_1 - i_1R_1 - i_1R_3 + i_2R_3 = 0$$

$i_1(R_1 + R_3) - i_2R_3 = V_1 \quad \boldsymbol{Eq.(1)}$

For mesh 2, applying **KVL** gives:

$-i_2R_2 - R_3(i_2 - i_1) - V_2 = 0, \qquad -i_2R_2 - i_2R_3 + i_1R_3 - V_2 = 0$

$i_1R_3 - i_2(R_2 + R_3) = V_2 \quad \boldsymbol{Eq.(2)}$

- The third step is to solve for the mesh currents. Putting Eqs. (1) and (2) in matrix form yields:

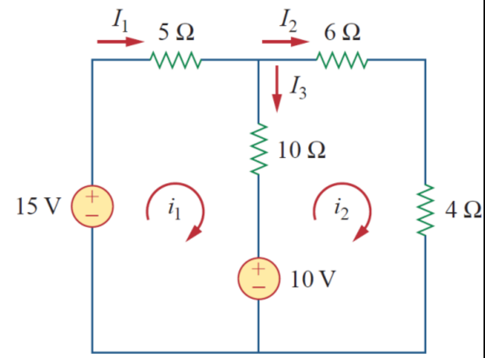
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ R_3 & -(R_2 + R_3) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Which can be solved to obtain the mesh currents $\boldsymbol{i_1}$and $\boldsymbol{i_2}$. After finding the mesh current:

$\boldsymbol{I_1 = i_1, I_2 = i_2, \;\; I_3 = i_1 - i_2}$

For the circuit in Fig. 3.1[illegible] d the branch currents I_1, I_2, and I_3 using mesh analysis.

Example 3.5



We first obtain the mesh currents using **KVL**:

For loop (1):

$$15 - 5i_1 - 10(i_1 - i_2) - 10 = 0, 15 - 5i_1 - 10i_1 + 10i_2 - 10 = 0$$

$$15i_1 - 10i_2 = 5, \quad \boldsymbol{3i_1 - 2i_2 = 1} \quad \boldsymbol{Eq. (1)}$$

For loop (2):

$$-6i_2 - 4i_2 - 10(i_2 - i_1) + 10 = 0, -10i_2 - 10i_2 + 10i_1 + 10 = 0$$

$$-20i_2 + 10i_1 = -10, \quad 10i_1 - 20i_2 = -10, \quad \boldsymbol{i_1 - 2i_2 = -1} \quad \boldsymbol{Eq. (2)}$$

We will use Cramer's rule to solve these two equations:

$\begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, We now obtain $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$ as:

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 1 & -2 \\ -1 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 1 & -2 \end{vmatrix}} = \frac{-2-2}{-6+2} = \frac{-4}{-4} = \boldsymbol{1A}$$

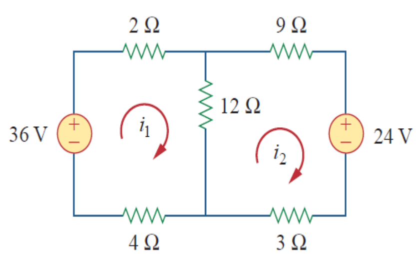
$$i_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix}}{-4} = \frac{-3-1}{-4} = \frac{-4}{-4} = \boldsymbol{1A}$$

$$I_1 = i_1 = \boldsymbol{1A}, \quad I_2 = i_2 = \boldsymbol{1A}, \quad I_3 = i_1 - i_2 = 1 - 1 = \boldsymbol{0A}$$

Practice Problem 3.5

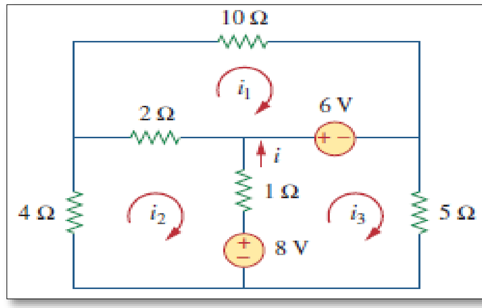
Calculate the mesh currents i_1 and i_2 of the circuit of Fig. 3.19.

Answer: $i_1 = 2$ A, $i_2 = 0$ A.



Example 3.7

Apply mesh analysis to find $\boldsymbol{i}$ in the circuit shown.



Solution:

For loop (1):

$$-10i_1 + 6 - 2(i_1 - i_2) = 0, -10i_1 + 6 - 2i_1 + 2i_2 = 0$$

$$12i_1 - 2i_2 = 6, \qquad \boldsymbol{6i_1 - i_2 = 3} \qquad \boldsymbol{Eq.(1)}$$

For loop (2):

$$-4i_2 - 2(i_2 - i_1) - 1(i_2 - i_3) - 8 = 0, -4i_2 - 2i_2 + 2i_1 - i_2 + i_3 - 8 = 0$$

$$-7i_2 + 2i_1 + i_3 = 8, \quad \boldsymbol{2i_1 - 7i_2 + i_3 = 8} \qquad \boldsymbol{Eq.(2)}$$

For loop (3):

$$-5i_3 + 8 - 1(i_3 - i_2) - 6 = 0, -5i_3 + 8 - i_3 + i_2 - 6 = 0$$

$$\boldsymbol{i_2 - 6i_3 = -2} \qquad \boldsymbol{Eq.(3)}$$

$$\begin{bmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ -2 \end{bmatrix}$$

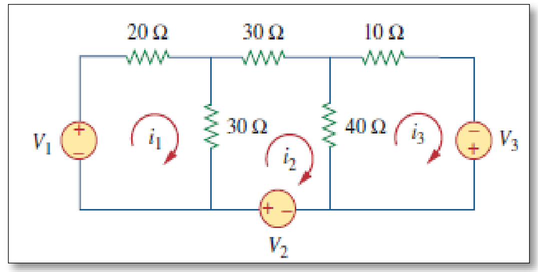
$$i_2 = \frac{\Delta_2}{\Delta} = \frac{\left| \begin{matrix} 6 & 3 & 0 \\ 2 & 8 & 1 \\ 0 & -2 & -6 \end{matrix} \right| \begin{matrix} 6 & 3 \\ 2 & 8 \\ 0 & -2 \end{matrix}}{\left| \begin{matrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & 1 & -6 \end{matrix} \right| \begin{matrix} 6 & -1 \\ 2 & -7 \\ 0 & 1 \end{matrix}} = \frac{-288 + 0 + 0 - 0 + 12 + 36}{252 + 0 + 0 - 0 - 6 - 12} = \boldsymbol{-1.025A}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{\left| \begin{matrix} 6 & -1 & 3 \\ 2 & -7 & 8 \\ 0 & 1 & -2 \end{matrix} \right| \begin{matrix} 6 & -1 \\ 2 & -7 \\ 0 & 1 \end{matrix}}{234} = \frac{84 + 0 + 6 - 0 - 48 - 4}{234} = \boldsymbol{0.136A}$$

$$i = i_2 - i_3 \ or \ i = i_3 - i_2 = -1.025 - 0.136 = \boldsymbol{-1.161A} \ or \ i = \boldsymbol{1.161A}$$

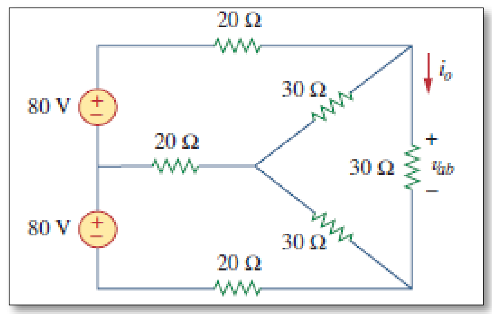
There is no need to find $\boldsymbol{i_1}$.

Practice Problem 3.6 Apply mesh analysis to find $\boldsymbol{i_1}, \boldsymbol{i_2}$ and $\boldsymbol{i_3}$ in the circuit shown if $V_1 = \mathbf{10}\boldsymbol{V}, V_2 = \mathbf{20}\boldsymbol{V}, and\ V_3 = \mathbf{30}\boldsymbol{V}.$



Answer: $\boldsymbol{i_1} = \mathbf{0.8}\boldsymbol{A},\ \boldsymbol{i_2} = \mathbf{1}\boldsymbol{A},\ \boldsymbol{i_3} = \mathbf{1.4}\boldsymbol{A}$

Example 3.8 Use mesh analysis to find $\boldsymbol{v_{ab}}$ and $\boldsymbol{i_0}$ in the circuit shown.



Solution:

For loop (1):

$80 - 20i_1 - 30(i_1 - i_3) - 20(i_1 - i_2) = 0$

$80 - 20i_1 - 30i_1 + 30i_3 - 20i_1 + 20i_2 = 0, 80 - 70i_1 + 20i_2 + 30i_3 = 0$

$\mathbf{70}\boldsymbol{i_1} - \mathbf{20}\boldsymbol{i_2} - \mathbf{30}\boldsymbol{i_3} = \mathbf{80} \qquad \boldsymbol{Eq.}(\mathbf{1})$

For loop (2):

$80 - 20(i_2 - i_1) - 30(i_2 - i_3) - 20i_2 = 0$

$80 - 20i_2 + 20i_1 - 30i_2 + 30i_3 - 20i_2 = 0, 80 + 20i_1 - 70i_2 + 30i_3 = 0$

$\mathbf{20}\boldsymbol{i_1} - \mathbf{70}\boldsymbol{i_2} + \mathbf{30}\boldsymbol{i_3} = -\mathbf{80} \qquad \boldsymbol{Eq.}(\mathbf{2})$

$-30(i_3 - i_1) - 30i_3 - 30(i_3 - i_2) = 0$

$-30i_3 + 30i_1 - 30i_3 - 30i_3 + 30i_2 = 0, \mathbf{30}\boldsymbol{i_1} + \mathbf{30}\boldsymbol{i_2} - \mathbf{90}\boldsymbol{i_3} = \mathbf{0} \qquad \boldsymbol{Eq.}(\mathbf{3})$

$$\begin{bmatrix} 70 & -20 & -30 \\ 20 & -70 & 30 \\ 30 & 30 & -90 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 80 \\ -80 \\ 0 \end{bmatrix}$$

No need to find $\boldsymbol{i_1}$ *and* $\boldsymbol{i_2}$.

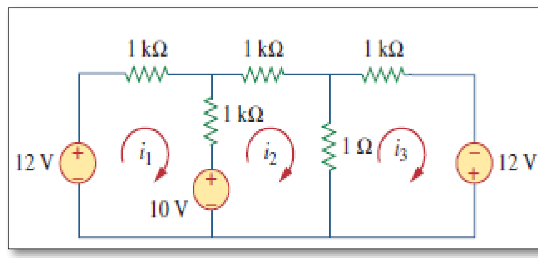
$$i_3 = \frac{\begin{vmatrix} 70 & -20 & 80 \\ 20 & -70 & -80 \\ 30 & 30 & 0 \end{vmatrix}\begin{matrix} 70 & -20 \\ 20 & -70 \\ 30 & 30 \end{matrix}}{\begin{vmatrix} 70 & -20 & -30 \\ 20 & -70 & 30 \\ 30 & 30 & -90 \end{vmatrix}\begin{matrix} 70 & -20 \\ 20 & -70 \\ 30 & 30 \end{matrix}}$$

$$i_3 = \frac{0 + 48000 + 48000 + 168000 + 168000 - 0}{441000 - 18000 - 18000 - 63000 - 63000 - 36000} = \boldsymbol{1.778A}$$

$$i_0 = i_3 = \boldsymbol{1.778A}, \quad v_{ab} = i_0 \times 30 = 1.778 \times 30 = \boldsymbol{53.34V}$$

Practice Problem 3.7

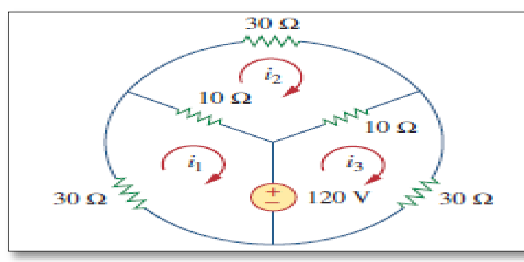
Find the mesh currents $\boldsymbol{i_1}, \boldsymbol{i_2}$ and $\boldsymbol{i_3}$ in the circuit shown.



Answer: $\boldsymbol{i_1 = 5.25mA, i_2 = 8.5mA, i_3 = 10.25mA}$

Practice Problem 3.9

Find the mesh currents $\boldsymbol{i_1}, \boldsymbol{i_2}$ *and* $\boldsymbol{i_3}$ in the circuit shown.

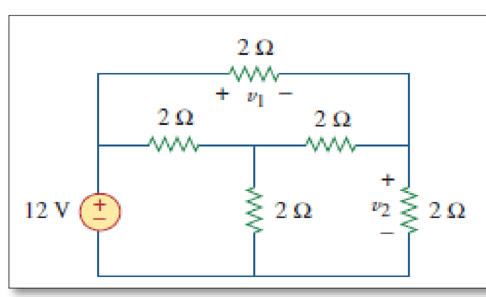


Answer: $\boldsymbol{i_1 = -3A, i_2 = 0A, i_3 = 3A}$

Practice Problem 3.9

Determine $\boldsymbol{v_1}$ *and* $\boldsymbol{v_2}$ in the circuit shown using mesh analysis.

Answer: $\boldsymbol{v_1 = 6V, v_2 = 6V}$



References:-

- Fundamentals of Electric Circuits, Charles K. Alexander, Matthew N. O. Sadiku, fourth edition.
- Electrical and Electronic Principles and Technology,JOHN BIRD, Second edition.
- Electrical Circuit Theory and Technology, JOHN BIRD, Second edition.

4

Chapter

Circuit Theorems

4.1 Introduction

A major advantage of analyzing circuits using Kirchhoff's laws as we did in Chapter 3 is that we can analyze a circuit without tampering with its original configuration. A major disadvantage of this approach is that, for a large, complex circuit, tedious computation is involved. The growth in areas of application of electric circuits has led to an evolution from simple to complex circuits. To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis. Such theorems include Thevenin's and Norton's theorems. Since these theorems are applicable to *linear* circuits, we first discuss the concept of circuit linearity. In addition to circuit theorems, we discuss the concepts of superposition, source transformation, and maximum power transfer in this chapter. The concepts we develop are applied in the last section to source modeling and resistance measurement.

4.2 Superposition

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis as in Chapter 3. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the *superposition.* The idea of superposition rests on the linearity property.

- Superposition is not limited to circuit analysis but is applicable in many fields where cause and effect bear a linear relationship to one another.
- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

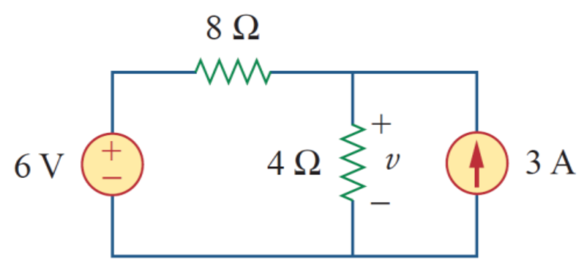
The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition
principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are *turned off*. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit.
2. Dependent sources are left intact because they are controlled by circuit variables. With these in mind, we apply the superposition principle in three steps:

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

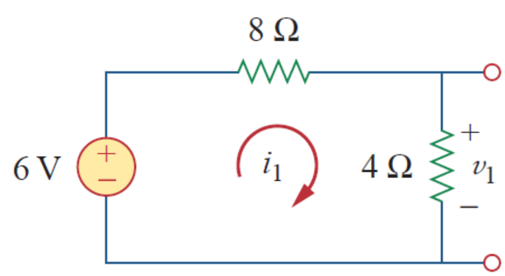
Example: Use the superposition theorem to find $\boldsymbol{v}$ in the circuit shown.



Solution:

The first step is to replace the current source by an open circuit as shown in Fig.(a):

$$v_1 = \frac{6 \times 4}{8+4} = \boldsymbol{2V}$$

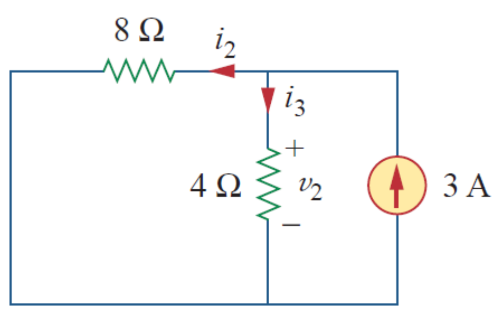


(a)

The second step is to replace the voltage source by a short circuit as shown in Fig. (b):

$$i_3 = \frac{3 \times 8}{4+8} = 2A, \quad v_2 = 2 \times 4 = \boldsymbol{8V},$$

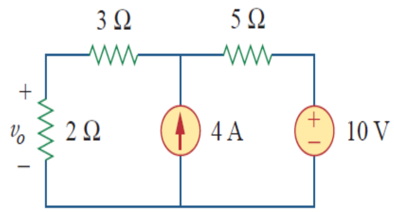
$$\boldsymbol{v = v_1 + v_2 = 2 + 8 = 10V}$$



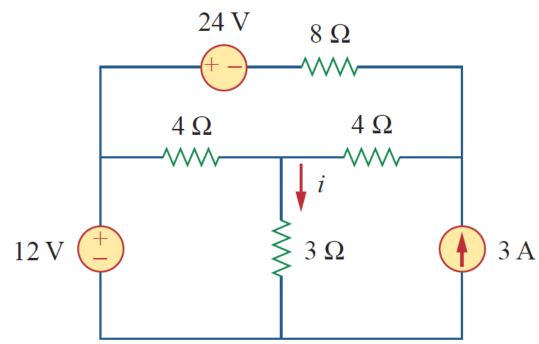
(b)

Practice Problem 4.3

Using the superposition theorem, find v_o in the circuit of Fig. 4.8.

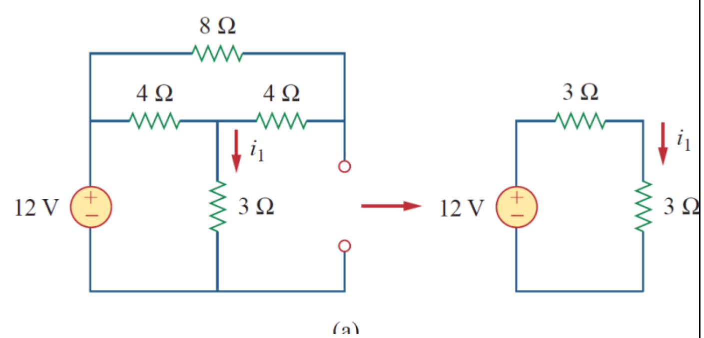


Answer: 6 V.



Example: For the circuit shown, use the superposition theorem to find $\boldsymbol{i}$.

The current due to the voltage source **12V**:

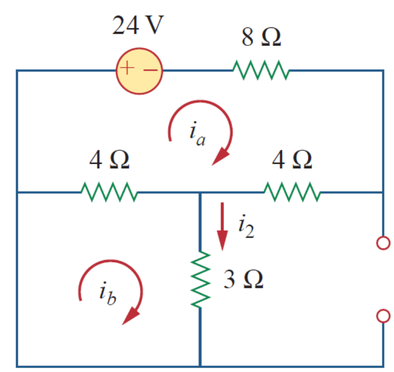


The resistors 4Ω on the right and 8Ω are in series:

$4 + 8 = 12\Omega$, The 12Ω in parallel with 4Ω gives: $\frac{4\times12}{4+12} = 3\Omega$

$i_1 = \frac{12}{6} = \boldsymbol{2A}$

The current due to the voltage source **24V**:



(b)

Applying mesh analysis gives:

For loop (1):

$$-24 - 8i_a - 4i_a - 4(i_a - i_b) = 0$$

$$\boldsymbol{4i_a - i_b = -6} \quad \boldsymbol{Eq.(1)}$$

For loop (2):

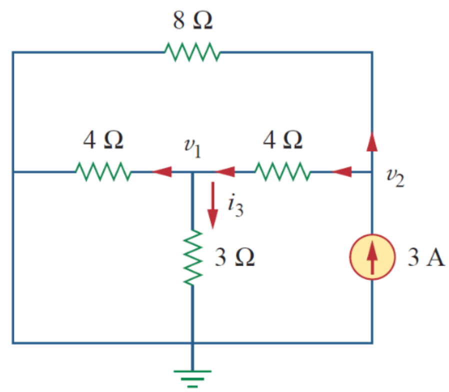
$$-4(i_b - i_a) - 3i_b = 0, \quad \boldsymbol{4i_a - 7i_b = 0} \quad \boldsymbol{Eq.(2)}$$

$$\begin{bmatrix}4 & -1\\4 & -7\end{bmatrix}\begin{bmatrix}i_a\\i_b\end{bmatrix} = \begin{bmatrix}-6\\0\end{bmatrix}, \quad i_b = \frac{\begin{vmatrix}4 & -6\\4 & 0\end{vmatrix}}{\begin{vmatrix}4 & -1\\4 & -7\end{vmatrix}} = \frac{0+24}{-28+4} = \boldsymbol{-1A = i_2}$$

The current due to the current source **3A**:

Using nodal analysis gives:

For node (1):



(c)

$$\frac{v_1 - v_2}{4} + \frac{v_1 - 0}{3} + \frac{v_1 - 0}{4} = 0$$

$$v_1\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{4}\right) - \frac{v_2}{4} = 0,$$

$$v_1\left(\frac{10}{12}\right) - v_2\left(\frac{1}{4}\right) = 0, \mathbf{10v_1 - 3v_2 = 0} \quad \boldsymbol{Eq.(1)}$$

For node (2):

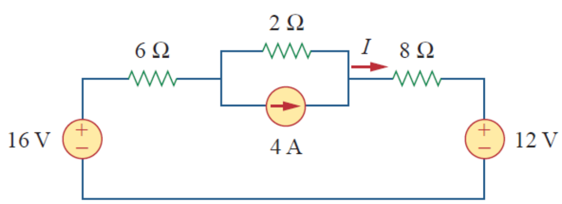
$$\frac{v_2 - v_1}{4} + \frac{v_2 - 0}{8} = 3, \quad v_2\left(\frac{1}{4} + \frac{1}{8}\right) - v_1\left(\frac{1}{4}\right) = 3, \mathbf{2v_1 - 3v_2 = -24} \quad \boldsymbol{Eq.(2)}$$

$$\begin{bmatrix} 10 & -3 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -24 \end{bmatrix}, v_1 = \frac{\begin{vmatrix} 0 & -3 \\ -24 & -3 \end{vmatrix}}{\begin{vmatrix} 10 & -3 \\ 2 & -3 \end{vmatrix}} = \frac{0 - 72}{-30 + 6} = \mathbf{3V}$$

There is no need to find v_2.

$$i_3 = \frac{v_1}{3} = \frac{3}{3} = \mathbf{1A}, \quad thus\ i = i_1 + i_2 + i_3 = 2 - 1 + 1 = \mathbf{2A}$$

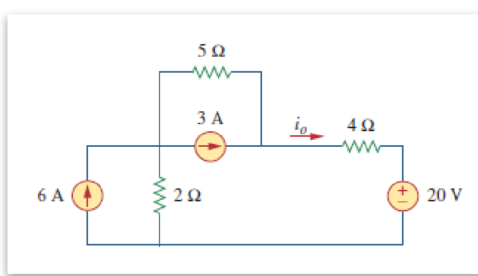
Find I in the circuit of Fig. using the superposition principle. **Practice Problem**



Answer: 0.75 A.

Practice Problem: For the circuit shown, use superposition theorem to find $\boldsymbol{i_0}$.

Answer: $\boldsymbol{i_0 = 0.636A}$.

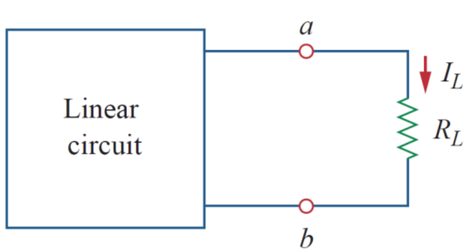


4.3 Thevenin's Theorem

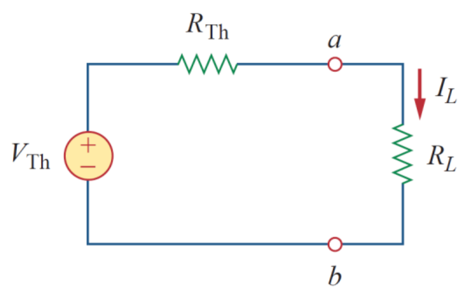
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

Steps to Find Thevenin's Equivalent Circuit:

1. Remove the load $\boldsymbol{R_L}$ from the circuit terminals a and b (The load may be a single resistor or another circuit) and redraw the circuit. The two terminals (a and b) have becomes open-circuited.
2. Calculate $\boldsymbol{R_{Th}}$ by first setting all independent sources to zero (voltage sources are replaced by short circuits and current sources are replaced by open circuits), and finding the resultant resistance between the network terminals.
3. Calculate $\boldsymbol{E_{Th}}$by first returning all sources to their original positions and finding the open circuit voltage between the network terminals.
4. Draw the **Thevenin's equivalent circuit** with $\boldsymbol{R_L}$ from where it was previously removed.
5. Finally, calculate the current flowing through the $\boldsymbol{R_L}$ by the following equation:



(a)

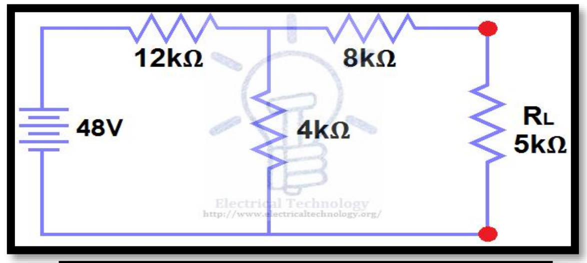


(b)

$$\boldsymbol{I_L = \frac{V_{Th}}{R_L + R_{Th}}}$$

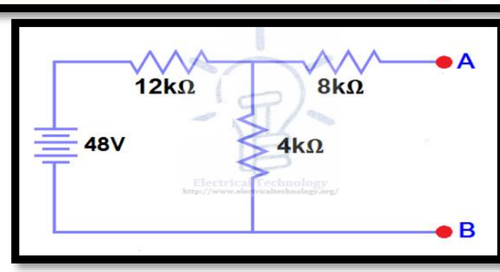
$$\boldsymbol{V_L = I_L R_L = (\frac{R_L}{R_L + R_{Th}}) V_{Th}}$$

Example: Using Thevenin's theorem, find the current in the $\boldsymbol{R_L}$ of the network shown.

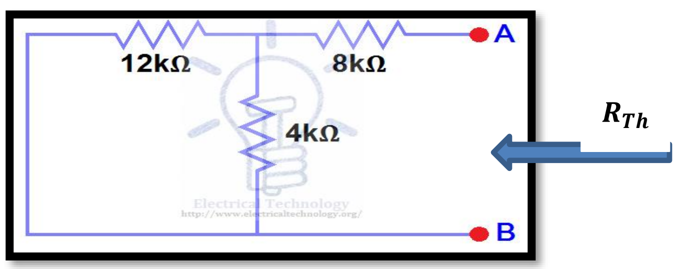


Solution:

Step (1): Remove $\boldsymbol{R_L}$ as shown.

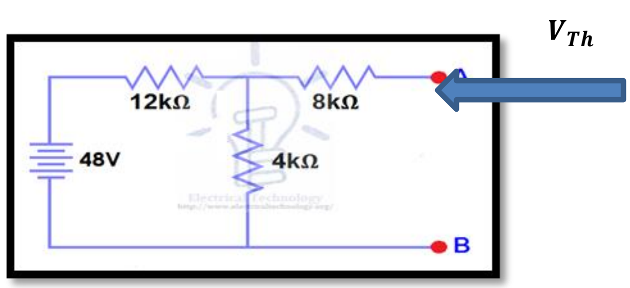


Step (2): Find $\boldsymbol{R_{Th}}$, remove the independent sources (replace the voltage source by short circuit (S. C.) as shown.



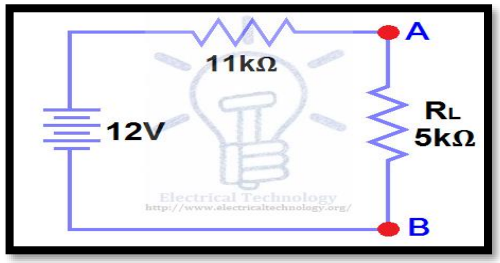
$$R_{Th} = (12 \| 4) + 8 = \frac{12 \times 4}{12 + 4} + 8 = 3 + 8 = \mathbf{11}\boldsymbol{k}\Omega$$

Step (3): Find $\boldsymbol{V_{Th}}$, return all sources to their original positions then determine V_{Th} using any method discussed previously across the open circuit terminals a and b as shown.



$$V_{Th} = \frac{48 \times 4 \times 1000}{(12 + 4) \times 1000} = \mathbf{12}\boldsymbol{V}$$

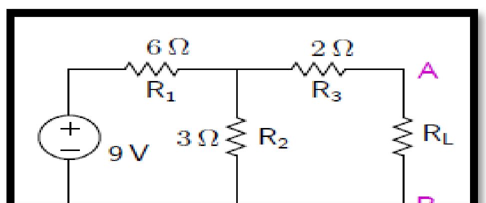
Step (4): Draw the **Thevenin's equivalent circuit** representing the network between points a and b with $\boldsymbol{R_L}$ added.



Step (5): Calculate the current flowing through the $\boldsymbol{R_L}$ as shown.

$$I_L = \frac{V_{Th}}{R_L + R_{Th}} = \frac{12}{11+5} = \boldsymbol{0.75mA}$$

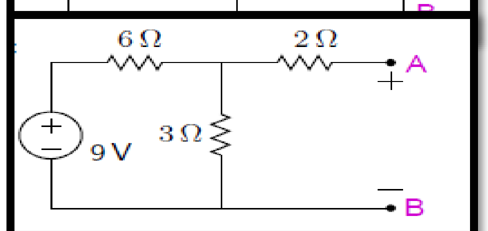
Example: Using Thevenin's theorem, find the current in the $\boldsymbol{R_L} = \mathbf{10}\Omega$ of the network shown.

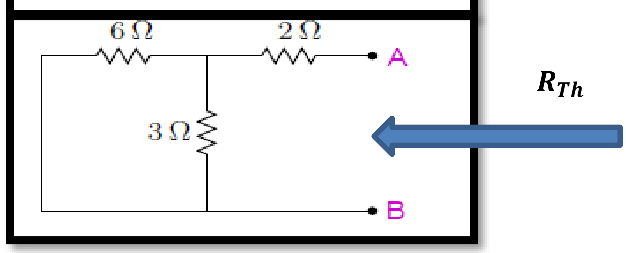


Solution:

Step (1):

Step (2):



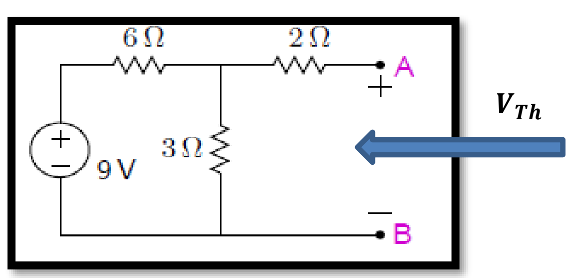


$$R_{Th} = (6 \| 3) + 2$$

$$= \frac{6 \times 3}{6+3} + 2$$

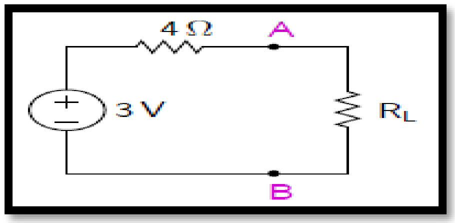
$$= \mathbf{4}\Omega$$

Step (3):



$$V_{Th} = \frac{9 \times 3}{3 + 6} = \boldsymbol{3V}$$

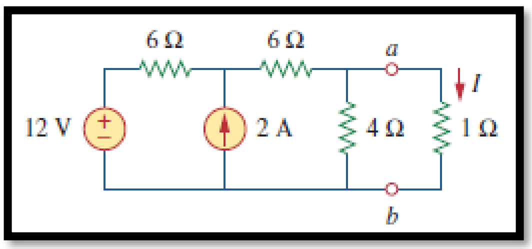
Step (4):



Step (5):

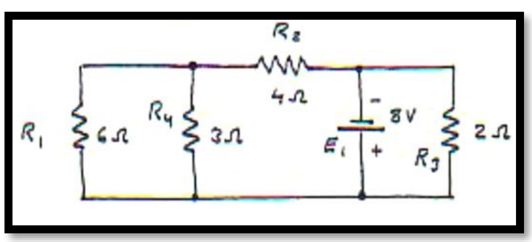
$$I_L = \frac{V_{Th}}{R_L + R_{Th}} = \frac{3}{4 + 10} = \boldsymbol{0.214A}$$

Practice Problem:Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown. Then find **I**.



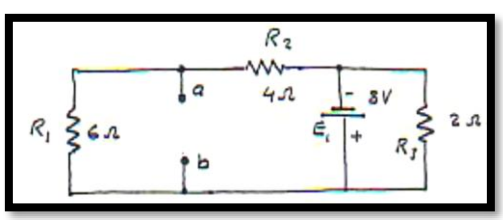
Answer: $\boldsymbol{V_{Th}} = \boldsymbol{6V}, \boldsymbol{R_{Th}} = 3\Omega, \boldsymbol{I} = \boldsymbol{1.5A}$.

Example: Find the current in the **3Ω** resistor using Thevenin's theorem in the circuit shown.

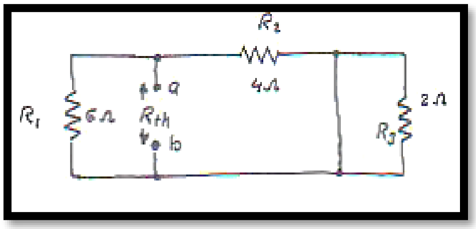


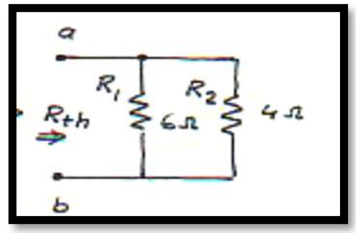
Solution:

Step (1):



Step (2):

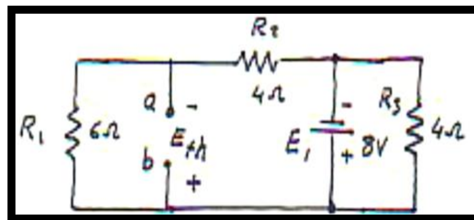




$$R_{Th} = 6 \| 4 = \frac{6 \times 4}{6+4} = \mathbf{2.4}\Omega,$$

R_3 *short circuited*

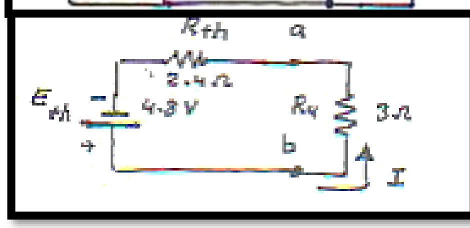
Step (3):



$$E_{Th} = \frac{8 \times 6}{6+4} = \mathbf{4.8V}$$

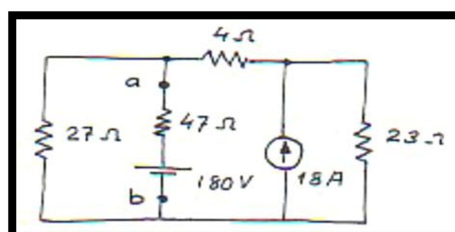
Step (4):

Step (5):

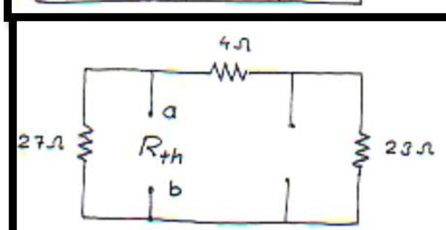


$$I = \frac{E_{Th}}{R_{Th} + R_4} = \frac{4.8}{2.4+3} = \mathbf{0.889A}$$

Example: In the circuit shown, find the current through the branch **a-b** using Thevenin's theorem.



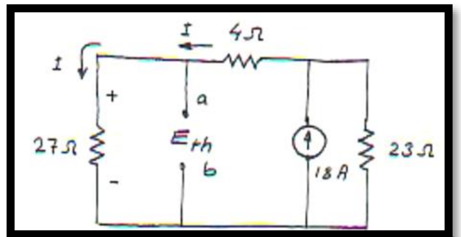
Solution:



$$R_{Th} = (4+23) \| 27 = 27 \| 27 = \frac{27 \times 27}{27+27} = \mathbf{13.5}\Omega$$

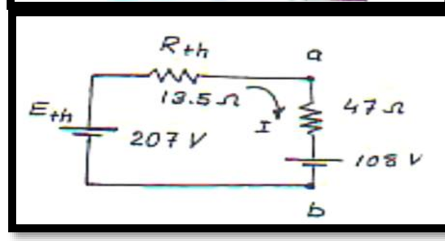
$$I = \frac{18 \times 23}{23+4+27} = 7.67A,$$

$E_{Th} = I(27) = 7.67 \times 27 = \mathbf{207.09V}$



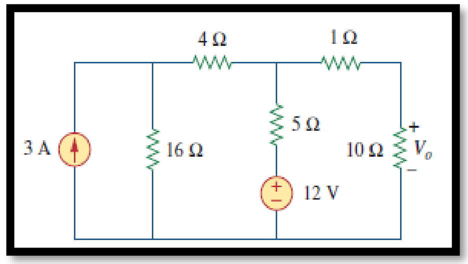
$I = \dfrac{E_{Th} - 108}{13.5 + 47}$

$$I = \frac{207.09 - 108}{13.5 + 47} = \mathbf{1.64A}$$

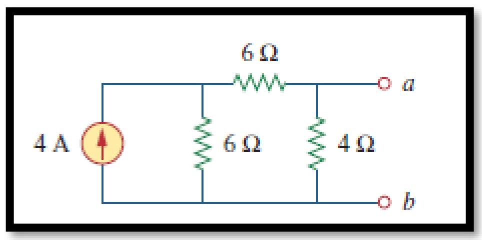


Practice Problem: Apply Thevenin's theorem to find $\boldsymbol{V_0}$ in the circuit shown.

Answer: $\boldsymbol{V_0 = 12.8V}$.



Practice Problem: Find the Thevenin's equivalent circuit to the left of the terminals a and b.



Answer: $\boldsymbol{R_{Th} = 3\Omega, V_{Th} = 6V}$.

References:-

- Fundamentals of Electric Circuits, Charles K. Alexander, Matthew N. O. Sadiku, fourth edition.
- Electrical and Electronic Principles and Technology,JOHN BIRD, Second edition.
- Electrical Circuit Theory and Technology, JOHN BIRD, Second edition.

5

Chapter

Capacitors and Inductors

5.1Introduction

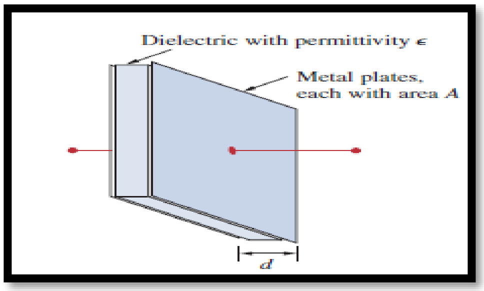
So far we have limited our study to resistive circuits. In this chapter, we shall introduce two new and important passive linear circuit elements: the capacitor and the inductor. Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called *storage* elements. The application of resistive circuits is quite limited. With the introduction of capacitors and inductors in this chapter, we will be able to analyze more important and practical circuits. Be assured that the circuit analysis techniques covered in Chapters 3 and 4 are equally applicable to circuits with capacitors and inductors. We begin by introducing capacitors and describing how to combine them in series or in parallel. Later, we do the same for inductors. As typical applications, we explore how capacitors are combined with op amps to form integrators, differentiators, and analog computers.

5.2 Capacitors

A capacitor is **a passive element** designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems.

A capacitor consists of two conducting plates separated by an insulator (or dielectric).

When a voltage source $\boldsymbol{v}$ is connected to the capacitor, the source deposits a positive charge $\boldsymbol{q}$ on one plate and a negative charge on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by $\boldsymbol{q}$, is directly proportional to the applied voltage so that:



$$\boldsymbol{q = Cv} \qquad \boldsymbol{Eq.(1)}$$

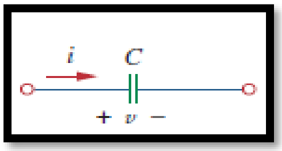
Where **C**, the constant of proportionality, is known as the capacitance of the capacitor.

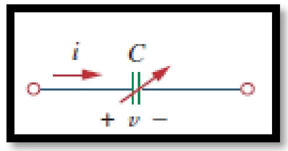
Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

Although the capacitance **C** of a capacitor is the ratio of the charge $\boldsymbol{q}$ per plate to the applied voltage $\boldsymbol{v}$ it does not depend on $\boldsymbol{q}$ or $\boldsymbol{v}$. It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown, the capacitance is given by:

$$\boldsymbol{C = \frac{\epsilon A}{d}} \qquad \boldsymbol{Eq.(2)}$$

Where **A** is the surface area of each plate, **d** is the distance between the plates, and $\boldsymbol{\epsilon}$ is the permittivity of the dielectric material between the plates. The circuit symbols for fixed and variable capacitors are shown in the figure below.





To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of Eq. (1). Since:

$$\boldsymbol{i = \frac{dq}{dt} \qquad Eq.(3)}$$

Differentiating both sides of Eq. (1) gives:

$$\boldsymbol{i = C\frac{dv}{dt} \qquad Eq.(4)}$$

The voltage-current relation of the capacitor can be obtained by integrating both sides of Eq. (4). We get:

$$\boldsymbol{v(t) = \frac{1}{C}\int_{t_1}^{t_2} idt \qquad Eq.(5)}$$

The instantaneous power delivered to the capacitor is:

$$\boldsymbol{P = vi = Cv\frac{dv}{dt} \qquad Eq.(6)}$$

The energy stored in the capacitor is therefore:

$$W = \int_{t_1}^{t_2} Pdt = \int_{t_1}^{t_2} vidt = \int_{t_1}^{t_2} Cv\frac{dv}{dt}dt = C\int_{t_1}^{t_2} vdv = C\frac{v^2}{2}$$

$$\boldsymbol{W = \frac{1}{2}Cv^2 \qquad Eq.(7)} \quad or \quad \boldsymbol{W = \frac{q^2}{2C} \qquad Eq.(8)}$$

We note from Eq. (4) that when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus,

A capacitor is an open circuit to dc.

Example: (a) Calculate the charge stored on a **3-pF** capacitor with **20 V** across it.

(b) Find the energy stored in the capacitor.

Solution:

$(a)\ q = Cv = 3 \times 10^{-12} \times 20 = \boldsymbol{60pC}$

$(b)\ W = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = \boldsymbol{600pJ}$

Example: The voltage across a **5-μF** capacitor is: $\boldsymbol{v(t) = 10cos6000t\ V}$, calculate the current through it.

Solution:

$$i = C\frac{dv}{dt} = 5 \times 10^{-6} \times \frac{d}{dt}(10cos6000t) = -5 \times 10^{-6} \times 6000 \times 10sin6000t$$

$i = \boldsymbol{-0.3sin6000t\ A}$

Q: What is the voltage across a $\boldsymbol{4.5 - \mu F}$ capacitor if the charge on one plate is **0.12 mC**? How much energy is stored?

Answer: 26.67 A, 1.6 mJ.

Q: If a **10-μF** capacitor is connected to a voltage source with:

$\boldsymbol{v(t) = 75sin2000t\ V}$, determine the current through the capacitor.

Answer: 1.5cos2000t A.

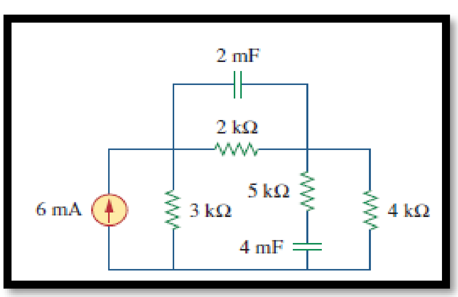
Example: Determine the voltage across a **2-μF** capacitor if the current through it is: $\boldsymbol{i(t) = 6e^{-3000t}mA}$, Assume that the initial capacitor voltage is zero.

Solution:

$$v = \frac{1}{C}\int_0^t idt = \frac{1}{2 \times 10^{-6}}\int_0^t 6e^{-3000t} \times 10^{-3}dt = \frac{3 \times 10^3}{-3000}e^{-3000t}\Big|_0^t$$

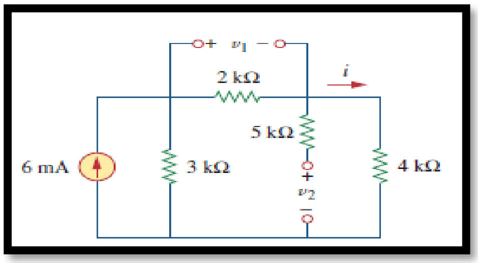
$\boldsymbol{v = (1 - e^{-3000t})V}$

Example: Obtain the energy stored in each capacitor in the circuit shown under dc conditions.



Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown.



The current through the series combination of the **2kΩ** and **4kΩ** resistors is obtained by current division as:

$$i = \frac{6 \times 3}{3 + 2 + 4} = \boldsymbol{2mA}$$

Hence, the voltages $\boldsymbol{v_1}$ *and* $\boldsymbol{v_2}$ *across the capacitors are*:

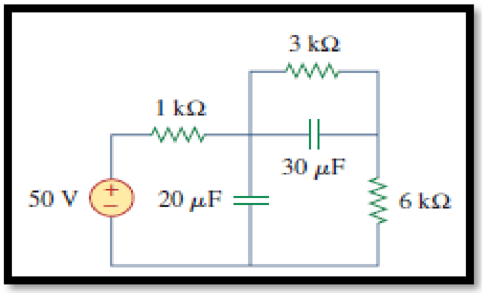
$$v_1 = 2000i = 2000 \times 2 \times 10^{-3} = \boldsymbol{4V}, v_2 = 4000i = \boldsymbol{8V}$$

And the energies stored in them are:

$$W_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}(2 \times 10^{-3}) \times (4)^2 = \boldsymbol{16mJ}$$

$$W_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}(4 \times 10^{-3}) \times (8)^2 = \boldsymbol{128mJ}$$

Q: Under dc conditions, find the energy stored in the capacitors in the circuit shown.



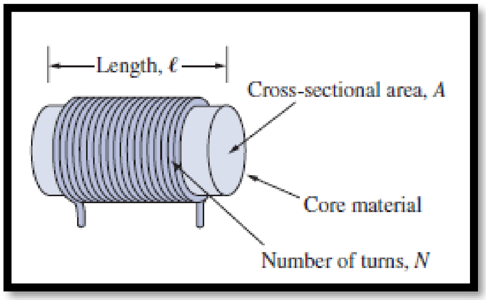
Answer: 20.25 mJ, 3.375 mJ.

5.3 Inductors

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems.

Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown.

An inductor consists of a coil of conducting wire.



If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current as shown:

$$v = L\frac{di}{dt} \qquad Eq.(1)$$

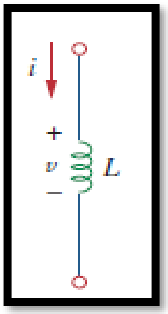
Where **L** is the constant of proportionality called the inductance of the inductor.

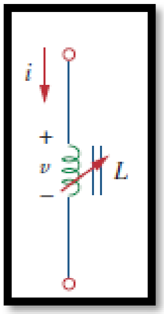
Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

The inductance of an inductor depends on its physical dimension and construction. For example, for the inductor, (solenoid) shown, the inductance is given by:

$$L = \frac{N^2 \mu A}{l} \qquad Eq.(2)$$

Where **N** is the number of turns, l is the length, **A** is the cross-sectional area, and μ is the permeability of the core. We can see from Eq. (2) that inductance can be increased by increasing the number of turns of coil, using material with higher permeability as the core, increasing the cross-sectional area, or reducing the length of the coil. The circuit symbols for fixed and variables inductors are shown in the figure below.





The current-voltage relationship is obtained from Eq. (1) as:

$$di = \frac{1}{L} v dt$$

Integrating gives:

$$\boldsymbol{i = \frac{1}{L} \int_{t_0}^{t} v dt \qquad Eq. (3)}$$

The inductor is designed to store energy in its magnetic field. The energy stored can be obtained from Eq. (1). The power delivered to the inductor is:

$$\boldsymbol{P = vi = \left(L \frac{di}{dt}\right) i \qquad Eq. (4)}$$

The energy stored is:

$$W = \int_{t_1}^{t_2} P dt = L \int_{t_1}^{t_2} \frac{di}{dt} i dt = L \int_{t_1}^{t_2} i di = L(\frac{i^2}{2})$$

$$\boldsymbol{W = \frac{1}{2} L i^2 \qquad Eq. (5)}$$

We note from Eq. (1) that the voltage across an inductor is zero when the current is constant. Thus,

An inductor acts like a short circuit to dc.

Example: The current through a **0.1-H** inductor is $\boldsymbol{i(t) = 10te^{-5t}}$**.** Find the voltage across the inductor and the energy stored in it.

Solution:

$$v = L \frac{di}{dt} = 0.1 \times \frac{d}{dt}(10te^{-5t}) = t \times (-5e^{-5t}) + e^{-5t} \times 1 = -5te^{-5t} + e^{-5t}$$

$$\boldsymbol{v = e^{-5t}(1 - 5t) V}$$

The energy stored:

$$W = \frac{1}{2} \times 0.1 \times (10te^{-5t})^2 = \frac{1}{2} \times 0.1 \times 100t^2 e^{-10t} = \boldsymbol{5t^2 e^{-10t} J}$$

Q: If the current through a 1-mH inductor is $\boldsymbol{i(t) = 60 cos 100t\ mA}$, find the terminal voltage and the energy stored.

Answer: -6 sin 100t mV, 1.8 $\boldsymbol{cos^2}$ (100t) $\boldsymbol{\mu}$J.

<u>Example</u>: Find the current through a **5-H** inductor if the voltage across it is:

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0 & t < 0 \end{cases}$$

Also, find the energy stored at t=5s. Assume $i(v) > 0$.

<u>Solution</u>:

$$i = \frac{1}{L}\int_{t_0}^{t} v dt = \frac{1}{5}\int_{0}^{t} 30t^2\, dt = 6 \times \frac{t^3}{3}\Big|_0^t = 2(t^3 - 0) = \boldsymbol{2t^3 A}$$

$$P = vi = 30t^2 \times 2t^3 = \boldsymbol{60t^5 W}$$

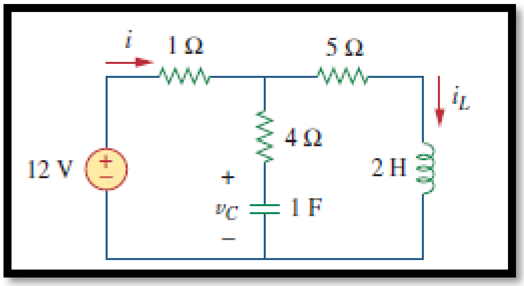
$$W = \int_0^t P dt = \int_0^5 60t^5\, dt = 60\frac{t^6}{6}\Big|_0^5 = 10(5^6 - 0) = \boldsymbol{156.25kJ}$$

Or $W = \frac{1}{2}Li^2 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li^2(0) = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 = \boldsymbol{156.25kJ}$

Q: The terminal voltage of a **2-H** inductor is $\boldsymbol{v = 10(1 - t)V}$. Find the current flowing through it at **t=4s** and the energy stored in it at **t=4s.** Assume $\boldsymbol{i(0) = 2A}$.

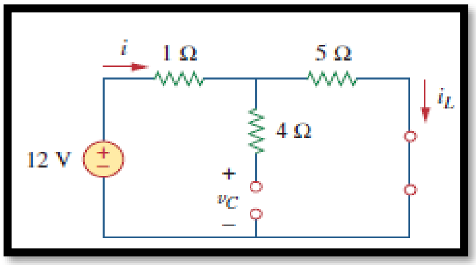
Answer: -18 A, 320 J.

<u>Example</u>: Consider the circuit shown. Under dc conditions, find: (a) $\boldsymbol{i}$, $\boldsymbol{v_c}$ and $\boldsymbol{i_L}$(b) the energy stored in the capacitor and inductor.



<u>Solution</u>:

(a) Under dc conditions, we replace the capacitor with an **open circuit** and the inductor with a **short circuit**, as shown:



$$i = i_L = \frac{12}{1+5} = \boldsymbol{2A}$$

The voltage $\boldsymbol{v_c}$ is the same as the voltage across the **5Ω** resistor. Hence:

$$v_c = 5i = 5 \times 2 = \boldsymbol{10V}$$

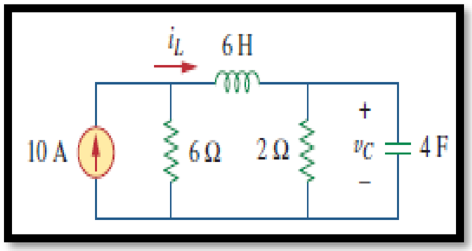
(b) The energy in the capacitor is:

$$W_c = \frac{1}{2}Cv_c^2 = \frac{1}{2} \times 1 \times (10)^2 = \boldsymbol{50J}$$

And that in the inductor is:

$$W_L = \frac{1}{2}Li_L^2 = \frac{1}{2} \times 2 \times (2)^2 = \boldsymbol{4J}$$

Q: Determine $\boldsymbol{v_c}, \boldsymbol{i_L}$ and the energy stored in the capacitor and inductor in the circuit shown under dc conditions.



Answer: 15 V, 7.5 A, 450 J, 168.75 J.

References:-

- Fundamentals of Electric Circuits, Charles K. Alexander, Matthew N. O. Sadiku, fourth edition.
- Electrical and Electronic Principles and Technology,JOHN BIRD, Second edition.
- Electrical Circuit Theory and Technology, JOHN BIRD, Second edition.
- Electronic Devices And Circuit Theory,Boylestad, 7th Edition.
- Electronic Devices And Circuit Theory,Boylestad, 9th Edition.

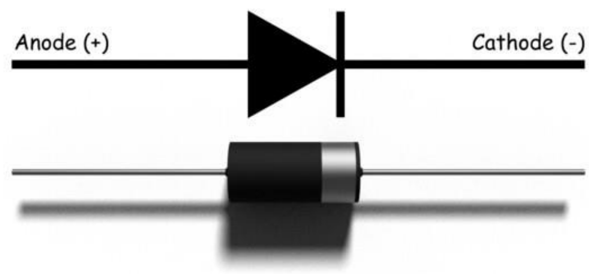
6

Chapter

Diode and Transistor

6.1 Diode:

A diode is an electric device that permits the flow of current only in one direction and restricts the flow in the opposite direction. The word “**diode**” is traditionally aloof for tiny signal appliances, **I ≤ 1 A**. When a diode is positioned in a simple battery lamp circuit, then the diode will either permit or stop flow of current through the lamp. There are various sorts of diode but their fundamental role is identical. The most ordinary kind of diode is silicon diode; it is placed in a glass cylinder.



❖ Diode Operation:

A diode starts its operations when a voltage signal applies across its terminals. A DC volt is applied so that diode starts its operation in a circuit and this is known as **Biasing**. Diode is similar to a **switch** which is one way, hence it can be either in conduction more or non-conduction mode. “**ON**” mode of the diode, is attained by **forward biasing**, which simply means that higher or positive potential is applied on the anode and on the cathode, negative or lower potential is applied of a diode. Whereas the “**OFF**” mode of the diode is attained with the aid of **reverse biasing** which simply means that higher or positive potential is applied on the cathode and on the anode, negative or lower potential is applied of a diode.

In the “**ON**” situation the practical diode provides forward resistance. A diode needs forward bias voltage to get in the “ON” mode this is known as **cut-in-voltage**. Whereas the diode initiates conducting in reverse biased manner when reverse bias voltage goes beyond its limit and this is known as **breakdown voltage**. The diode rests in **OFF** mode when no voltage is applicable across it.

❖ Function of Diode:

The key function of a diode is **to obstruct the flow of current in one direction and permit the flow of current in the other direction**. Current passing through the diode is known as **forward current** whereas the current blocked by the diode is known as the **reverse current**.

❖ **Diode Equation:**

The equation of diode expresses the current flow via diode as a function of voltage. The ideal diode equation is:

$$\boldsymbol{I = I_0(e^{\frac{qV}{KT}} - 1)}$$

Where:

I – Stands for the net current passing through a diode.

$\boldsymbol{I_0}$**-** Stands for dark saturation current, the diode seepage current density in the deficiency of light.

V – Stands for applied voltage across the terminals of the diode.

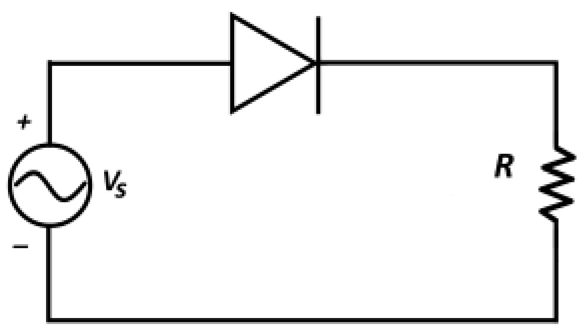
q – Stands for fixed value of electron charge.

k – Stands for Boltzmann's constant.

T – Stands for fixed temperature (K).

❖ **Diode Circuits:**

The basic aim behind this study is to show how diodes can be employed in circuits. Now let us analyze a simple diode circuit.



1. When diode is in **ON** mode, no voltage is there across it; hence it acts like a **short circuit**.
2. Whereas when diode is in **OFF** mode, there is zero current, hence it behaves like an **open circuit**.
3. From the above two conditions, either one can take place at a time. This helps us to check out what will happen in any circuit with diodes.

❖ **Diode Characteristics:**

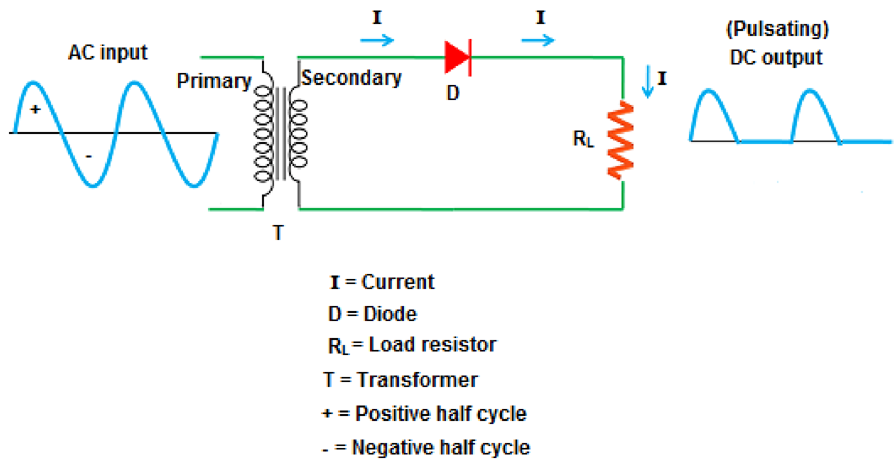
Diodes have attributes that allow them to carry out a number of electronic functions. Three vital characteristics of diodes are as follows:

- Forward Voltage Drop- forward bias about **seven volts**.
- Reverse Voltage Drop- Weakened layer broadens, generally the applied voltage.
- Reverse breakdown voltage- reverse voltage drop that'll force flow of current and in maximum cases demolish the diodes.

6.1.1 Application of Diodes:

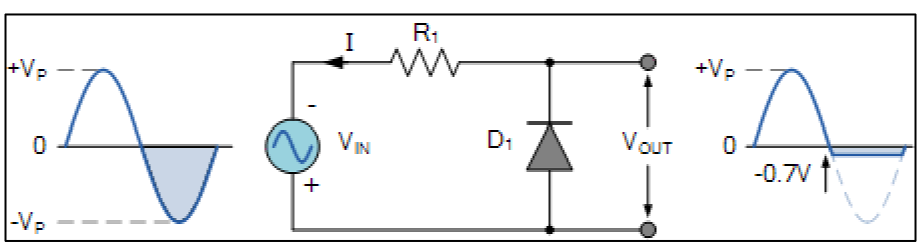
Diodes are employed in a variety of applications such as clipper, rectification, clamper, comparator, voltage multiplier, filters, sampling gates, etc.

1. **Rectification**: Rectification symbolizes the alteration of AC volt into DC volt. Some of the common examples of rectification circuits are- FWR (full wave rectifier), bridge rectifier & HWR (half wave rectifier).

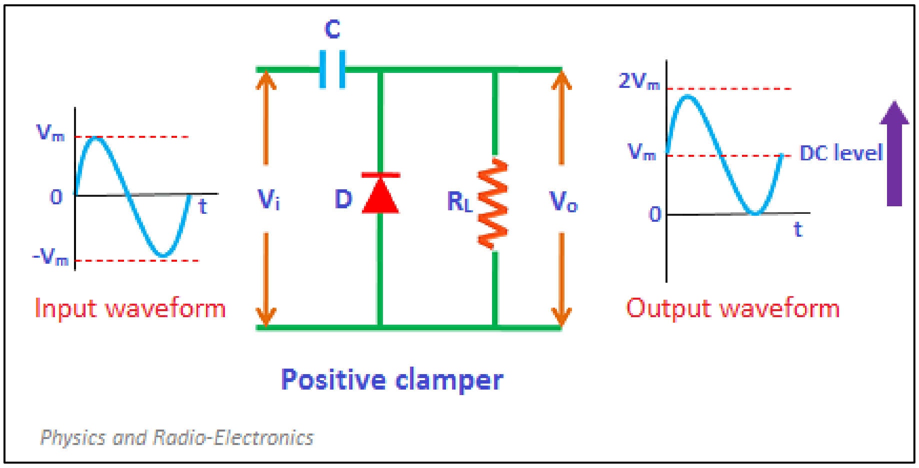


Half wave rectifier

2. **Clipper**: Diode can be employed to trim down some fraction of pulse devoid of deforming the left over fraction of the waveform.



3. **Clamper**: A clamping circuit limits the level of voltage to go beyond a limit by changing the DC level. The crest to crest is not influenced by clamping. Capacitors, resistors & diodes all are used to create clamping circuits.



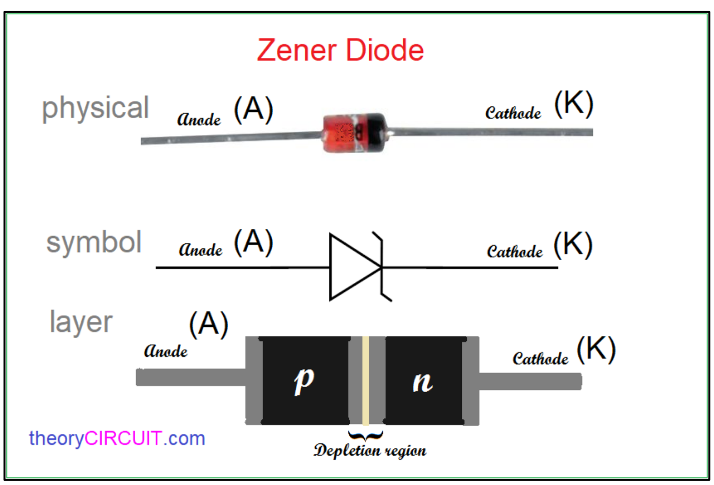
6.1.2 Types of Diodes:

All sort of diodes are dissimilar in means of construction, characteristics & applications. Following are some of the types of diodes:

- **Zener Diodes.**
- **LED (Light Emitting Diodes).**
- **Photodiodes.**
- **Shockley Diode.**
- **Tunnel Diodes.**
- **Varactor Diodes.**

Zener Diode:

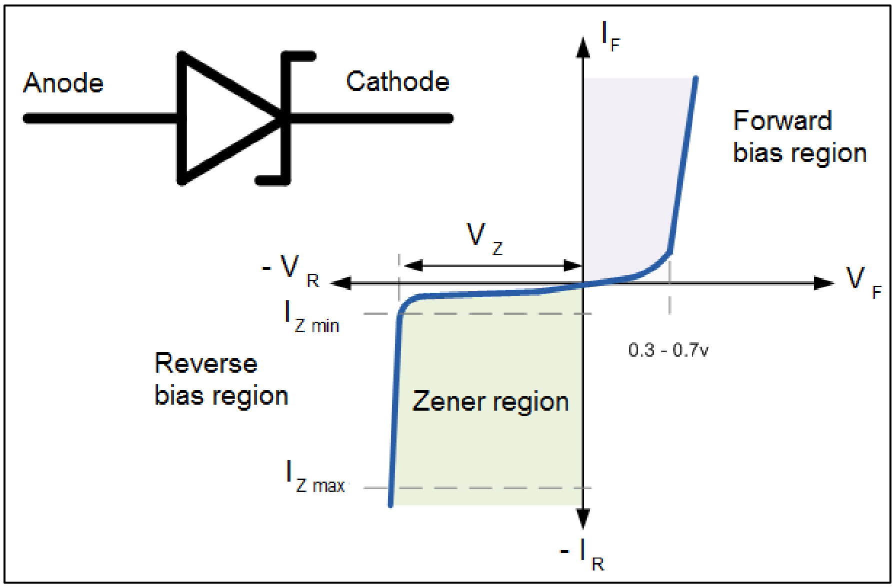
Zener diode works in **reverse bias situation** when the voltage attains the breakdown peak. An even voltage can be attained by insertion of a resistor across it to limit the flow of current. This Zener diode is employed to give reference voltage in power supplying circuits.



- **Zener Diode Characteristics**:

Special diodes such as zener diodes are intended & manufactured to function in the **opposite direction without being broken**.

1. The zener diode acts like a common silicon diode, during the forward bias.



2. Changeable quantity of reverse current can go through the diode devoid of destructing it. The **Vz** (zener voltage) or breakdown voltage across the diode upholds comparatively steady.
3. Producers rate zener diodes as per their zener voltage value and the highest PD (Power Dissipation) i.e. at 25ºC. This provides a signal of the highest IR (reverse current), that a diode securely carries out.

- **Zener Diode Applications:**

Zener diodes have many applications in transistor circuitry. Here we are discussing various vital points in Zener diode applications:

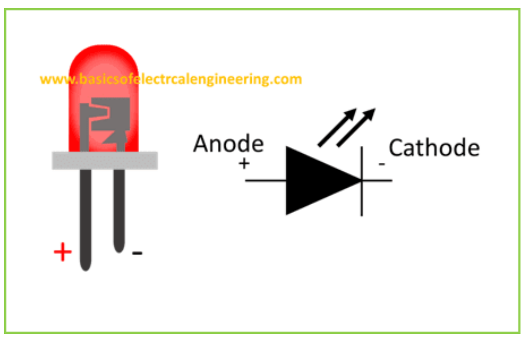
1. **Zener Diode Shunt Regulator**- This diode is commonly employed as a Voltage Regulator or Shunt Regulator.
2. **Meter Protection**- This diode may also come across its functions in meter security.
3. **Zener Diode as Peak Clipper**- This diodes can be employed to cut off the maximum value of incoming waveform.
4. **Switching operation**- This diode can generate an unexpected alteration from low to high current, so it is functional in switching applications. It is relatively speedy in switching processes.

Tunnel Diode:

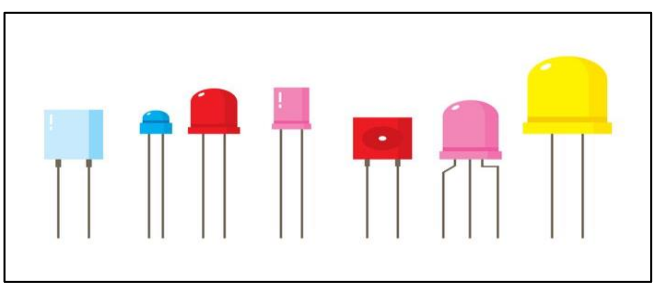
A tunnel diode is a highly conducting two terminal p-n jnction diode doped heavily approximately **1000 times** upper than a usual junction diode. A tunnel diode is also named as **Esaki diode**, it's named after Esaki diode who is a Nobel prize winner in physics for discovering electron tunneling outcome employed in these diodes. Tunnel diodes are helpful in several circuit purposes like in- microwave oscillation, binary memory & microwave amplification. Tunnel diodes are generally made-up from **gallium or germanium** or **gallium arsenide**. These all comprise tiny prohibited energy breaks and elevated ion motilities.

Light Emitting Diode or LED:

LED is a semiconductor appliance that **produces visible light beams or infrared light beams when an electric current is passed through it**. Visible LEDs can be seen in several electronic devices such as microwaves' number display light, brake lights, and even cameras to make use of Infrared LEDs. In LEDs (light emitting diodes) light is created by a solid situation procedure which is named as electroluminescence.



Light emitting diodes are available in various colors like- orange, red, yellow, amber, green, white & blue. Blue & white LEDs are more costly in comparison to other LEDs. The color of a Light emitting diodes is decided by the semi-conductor substance, not by the coloring the plastic of the body.



Varactor diode:

Varicap or Varactor diode is that shows the attributes of a **variable capacitor**. The exhaustion area at the p-n junction behaves as the di-electric and plates of an ordinary capacitor and grounds expansion and contraction by the voltage applied to the varicap diode. This action boosts and reduces the capacitance. The graphic symbol for the varicap diode is shown below.

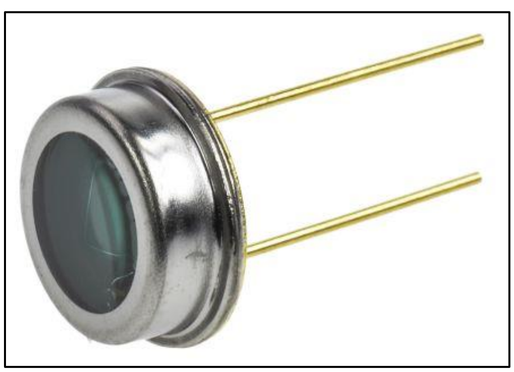
Varactors are employed in fine-tuning circuits and can be employed as high frequency amplifiers. Even though varactor or varicap diodes can be employed inside several sorts of circuit, they discover applications inside 2 key areas:-

1. RF filters.
2. Voltage controlled oscillators, VCOs.

Photo Diode:

Photo diodes are extensively used in various kinds of electronics such as detectors in compact disc players to optical telecommunications systems. Photo diode technology is popular because its trouble-free, inexpensive yet strong configuration. As photodiodes provide dissimilar properties, various photodiode technologies are utilized in a number of areas. There are 4 types of photo diodes:

1. PN photodiode.
2. Schottky photodiode.
3. Avalanche photodiode.
4. PIN photodiode.



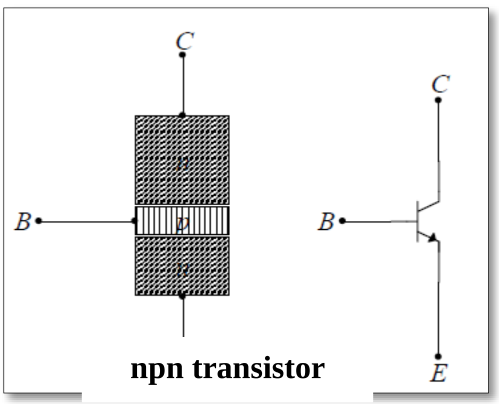
6.2 *Transistors:*

6.2.1 Bipolar Junction Transistors (BJT)

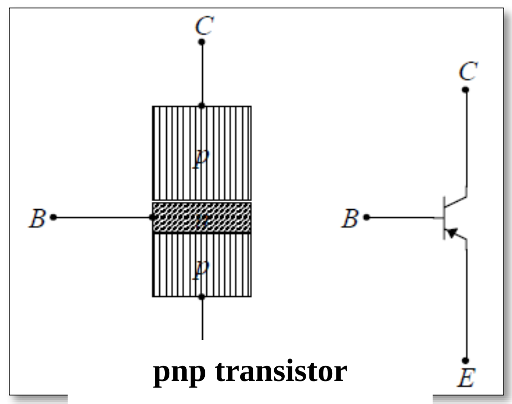
General configuration and definitions

The transistor is the main building block "element" of electronics. It is a semiconductor device and it comes in two general types: the Bipolar Junction Transistor (**BJT**) and the Field Effect Transistor (**FET**). Here we will describe the system characteristics of the BJT configuration and explore its use in fundamental signal shaping and amplifier circuits.

The BJT is a three terminal device and it comes in two different types. **The npn BJT and the pnp BJT**. The BJT symbols and their corresponding block diagrams are shown:



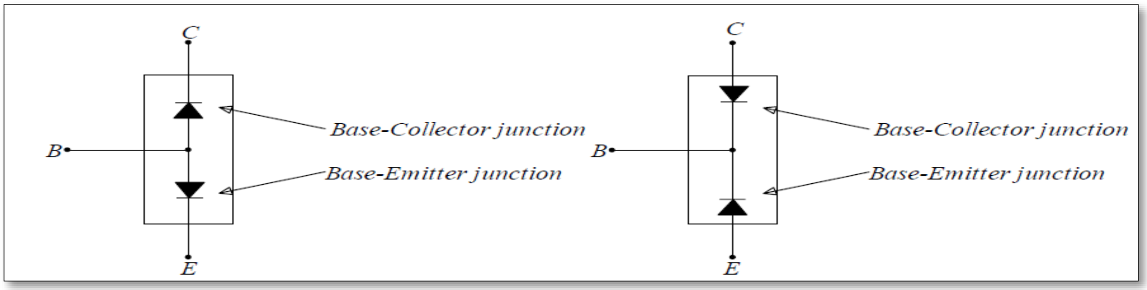
npn transistor



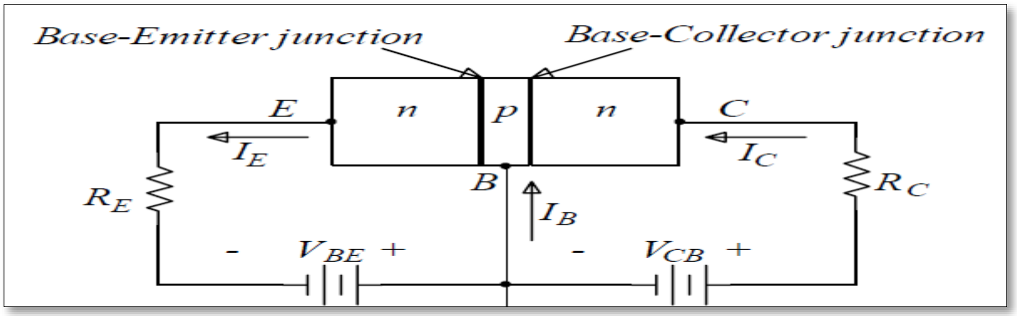
pnp transistor

The BJT is fabricated with three separately doped regions. The npn device has one p region between two n regions and the pnp device has one n region between two p regions. The BJT has two junctions (boundaries between the n and the p regions). These junctions are similar to the junctions we saw in the diodes and thus they may be forward biased or reverse biased. By relating these junctions to a diode model the pnp BJT may be modeled as shown:

The three terminals of the BJT are called the Base (**B**), the Collector (**C**) and the Emitter (**E**). Since each junction has two possible states of operation (forward or reverse bias) the BJT with its two junctions has four possible states of operation.



Before proceeding let's consider the BJT npn structure shown:



With the voltage $\boldsymbol{V_{BE}}$ and $\boldsymbol{V_{CB}}$ as shown, the Base-Emitter (**B-E**) junction is forward biased and the Base-Collector (**B-C**) junction is reverse biased. The current through the B-E junction is related to the B-E voltage as:

$$I_E = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$

Due to the large differences in the doping concentrations of the emitter and the base regions the electrons injected into the base region (from the emitter region) results in the emitter current $\boldsymbol{I_E}$. Furthermore the number of electrons injected into the collector region is directly related to the electrons injected into the base region from the emitter region.

Therefore, the collector current is related to the emitter current which is in turn a function of the B-E voltage.

<u>The voltage between two terminals controls the current through the third terminal.</u>

This is the basic principle of the BJT.

The collector current and the base current are related by:

$I_C = \beta I_B \qquad Eq.(1)$

And by applying **KCL** we obtain:

$I_E = I_B + I_C \qquad Eq.(2)$

And thus from equations (1) and (2) the relationship between the emitter and the base currents is:

$$I_E = I_B + I_C = I_B + \beta I_B = \boldsymbol{I_B(1+\beta)} \qquad \boldsymbol{Eq.(3)}$$

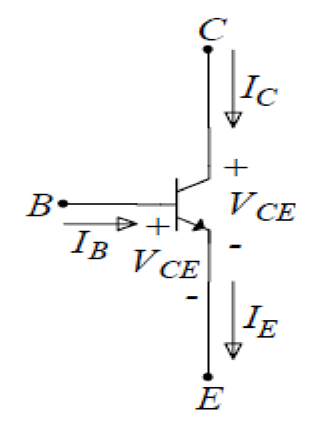
And equivalently:

$$I_B = \frac{I_E}{1+\beta}, \quad I_C = \beta \times \frac{I_E}{1+\beta} = \frac{\boldsymbol{\beta}}{\boldsymbol{1+\beta}}\boldsymbol{I_E} \qquad \boldsymbol{Eq.(4)}$$

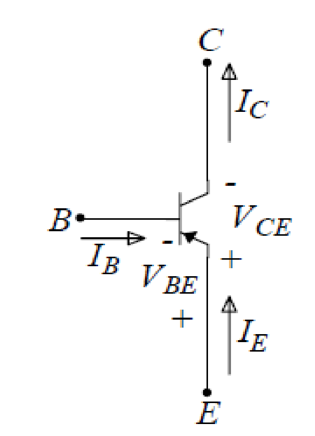
The fraction $\frac{\boldsymbol{\beta}}{\boldsymbol{1+\beta}}$ is called $\boldsymbol{\alpha}$.

For the transistors of interest $\boldsymbol{\beta = 100}$ which corresponds to $\boldsymbol{\alpha = 0.99}$ and $\boldsymbol{I_C \cong I_E}$.

The direction of the currents and the voltage polarities for the **npn** and the **pnp** BJTs are shown.



(a) ***npn*** transistor



(b) ***pnp*** transistor

Transistor Voltages:

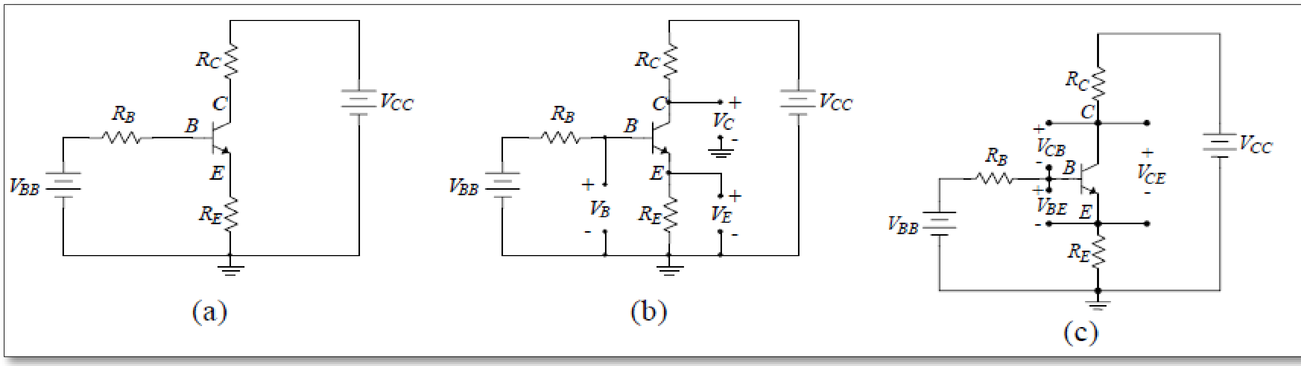
Three different types of voltages are involved in the description of transistors and transistor circuits. They are:

Transistor supply voltages: $\boldsymbol{V_{CC}, V_{BB}}$.

Transistor terminal voltages: $\boldsymbol{V_C, V_B, V_E}$.

Voltages across transistor junctions: $\boldsymbol{V_{BE}, V_{CE}, V_{CB}}$.

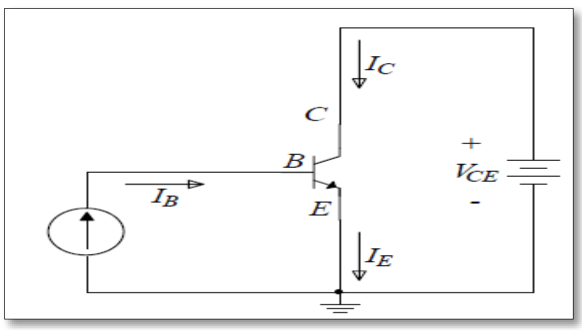
All of these voltages and their polarities are shown for the **npn BJT**.



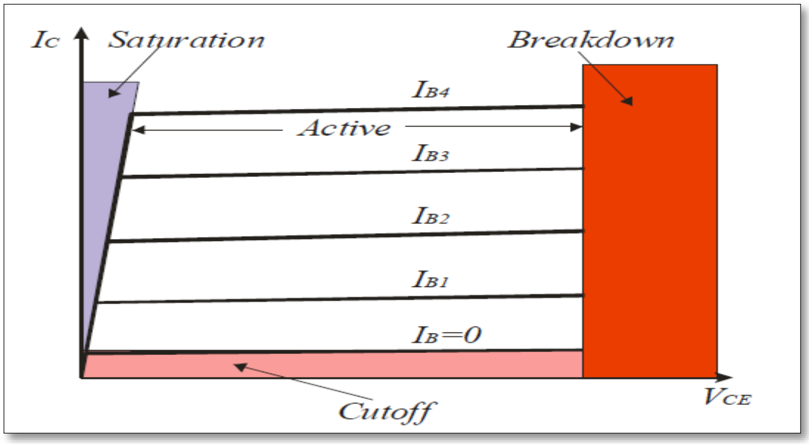
6.2.2 Transistor Operation and Characteristic i-v curves

The three terminals of the transistors and the two junctions, present us with multiple operating regimes. In order to distinguish these regimes we have to look at the **i-v** characteristics of the device.

The most important characteristic of the BJT is the plot of the collector current, $\boldsymbol{I_C}$, versus the collector-emitter voltage,$\boldsymbol{V_{CE}}$, for various values of the base current, $\boldsymbol{I_B}$ as shown on the circuit below:



The figure below shows the qualitative characteristic curves of a BJT. The plot indicates the four regions of operation: **the saturation**, **the cutoff**, **the active** and **the breakdown**. Each family of curves is drawn for a different base current and in this plot $\boldsymbol{I_{B4} > I_{B3} > I_{B2} > I_{B1}}$.



The characteristics of each region of operation are summarized below:

1. Cutoff region:

Base-emitter junction is reverse biased. No current flow.

2. Saturation region:

Base-emitter junction forward biased.

Collector-base junction is forward biased.

$\boldsymbol{I_C}$ Reaches a maximum which is independent of $\boldsymbol{I_B}$ and $\boldsymbol{\beta}$.

No control.

$\boldsymbol{V_{CE} < V_{BE}}$.

3. Active region:

Base-emitter junction forward biased.

Collector-base junction is reverse biased.

Control, $\boldsymbol{I_C = \beta I_B}$ (as can be seen from the fig., there is a small slope of $\boldsymbol{I_C}$ with $\boldsymbol{V_{CE}}$. $\boldsymbol{V_{BE} < V_{CE} < V_{CC}}$.

4. Breakdown region:

$\boldsymbol{I_C}$ and $\boldsymbol{V_{CE}}$ exceed specifications.

Damage to the transistor.

7 Basic BJT Applications:

1. As switch circuit.
2. Digital Logic. 3. Amplifier Circuit.

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